CHAPTER-8

APPLICATION OF INTEGRALS

01 MARK TYPE QUESTIONS

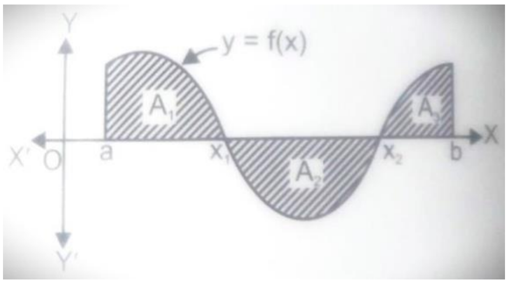
Q. NO	QUESTION	MARK
1.	The area of the region bounded by y = cos x between x=0 and x= π is a) 2sq unit b) 4sq unit c) 6sq unit d) 1sq unit	1
2.	The area of the region bounded by the parabola $y^2 = x$ and the straight line y = x/2 is a) 1/3 sq unit b) 2/3 sq unit c) 3/3 sq unit d) 4/3 sq unit	1
3.	The area bounded by the curve $y^2 = 4ax$ and axis between y= -a and y=a is a) A^2 b) $6a^2$ c) $A^2/6$ d) $A^2/2$	1
4.	The area of the region bounded by the curve x=2y+3 and the line y= 1and y= -1is a) 2 sq unit b) 4 sq unit c) 6 sq unit d) 8 sq unit	1
5.	The area bounded by the curve $y= x^2 -1$ and the straight line x + y=3 a) $\frac{\sqrt{17}}{7}$ sq unit b) $\frac{7\sqrt{17}}{6}$ sq unit c) $\frac{17\sqrt{17}}{6}$ sq unit d) 4 sq unit	1
6.	The area of the region bounded by y = |x-2| , x=1 and x=3 and x-axis is a) 4sq unit b) 3sq unit c) 2sq unit d) 1 sq unit	1
7.	Area of the triangle whose vertices formed from the x-axis and the line 3-|x| is a) $\frac{9}{2}$sq. unit b) $\frac{3}{2}$sq. unit c) $9 sq. unit$	1

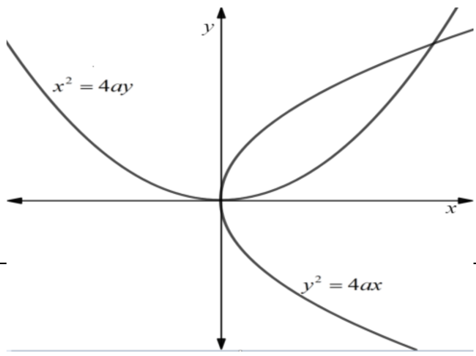
	d) 3 sq unit	
8.	Find the area of the region $\{(x,y): x^2 \le y \le x\}$. a) $\frac{1}{3}$sq. unit b) $\frac{1}{2}$sq. unit c) $\frac{1}{6}$sq. unit d) $\frac{1}{9}$sq. unit	1
9.	If y= 2 sin x + sin 2x for $0 \le x \le 2\pi$ then area enclosed by the curve and the x-axis is a) $\frac{9}{2}$sq. unit b) 8sq. unit c) 12sq. unit d) 4sq. unit	1
10.	The area of the region bounded by the curves y=x, x=e and y=$\frac{1}{x}$ and all the positive x-axis is a) $\frac{1}{2}$sq. unit b) $\frac{3}{2}$ sq. unit c) 1sq. unit d) $\frac{5}{2}$sq. unit	1
11.	The area of the region bounded by the circle $x^2 + y^2 = 1$ is (a) 2π sq. units (b) π sq. units (c) 3π sq. units (d) 4π sq. units	1
12.	The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is (a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units (c) $\frac{11}{2}$ sq. units (d) $\frac{13}{2}$ sq. units	1
13.	The area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is (a) 2 (b) $\frac{9}{4}$ (c) $\frac{9}{3}$ (d) $\frac{9}{2}$	1
14.	The area bounded by $y = 2 - x^2$ and $x + y = 0$ is (a) $\frac{7}{2}$ (b) $\frac{9}{2}$ (c)9 (d) none of these	1
15.	The area bounded by the parabola $x = 4 - y^2$ and y-axis, in square units, is (a) $\frac{3}{32}$ (b) $\frac{32}{3}$ (c) $\frac{33}{2}$ (d) $\frac{16}{3}$	1
16.	Area lying between the curve $y^2 = 4x$ and $y = 2x$ is (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$	1

17.	The area bounded by the parabola $y^2 = 4ax$, latus rectum and x-axis is (a) 0 (b) $\frac{4}{3}a^2$ (c) $\frac{2}{3}a^2$ (d) $\frac{a^2}{3}$	1
18.	The area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a) πab (b) $\pi a^2 b^2$ (c) $2\pi ab$ (d) ab	1
19.	The area of the region bounded by the circle $x^2 + y^2 = a^2$ (a) $2\pi a$ (b) πa^2 (c) $2\pi a^2$ (d) None of these	1
20.	The area of the region bounded by the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (a) 6π (b) 36π (c) 18π (d) None of these	1
21.	The area enclosed between the curve $y=x^2$ and $y=\sqrt{x}$ is A) ⅙ sq. unit B) ½ sq. unit C) 4 sq. unit D) 1/3 sq. unit	1
22.	The area enclosed among the curves 2x-3y=0, X axis, X=3 and X= 5 is A) 16 sq. units B) 8 sq. units C) 4 sd. units D) 16/3 sq. units	1
23.	Area bounded by the lines y=2+x, y= 2-x and x=2 is A) 3 sq. units B) 4 sq. units C) 8 sq. units D) 16sq. units	1
24.	Area lying in the first quadrant and bounded by the circle $x^2+y^2=4$, and the lines x=0 and x=2 is A) π B) π/4 C) π/3 D) π/2	1
25.	The area of the region bounded by the curve $y^2=4x$, Y axis and the line y=3 is A) 2 B)9/4 C)9/3 D) 9/2	1
26.	The area bounded by the curves $y^2=4ax$ and its latus rectum is A) 4/3 a^2 sq. Units B) 8/3 a^2 sq. Units C) 16/3a^2 sq. Units D)None of these	1
27.	Area bounded by the curve y= sinx between the ordinates x=0 and x=π is A) 2sq. Units B) 4 sq. Units C) 3 sq. Units D) 1 sq. Units	1
28.	Assertion (A): The area bounded by the circle x^2+y^2= 16 is 16π sq. Units. Reason (R): We have x^2+y^2= 16, which is circle having center at (0,0) and radius 4 units. (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A.	1

	(C) A is true but R is false. (D) A is false but R is true.	
29.	Assertion (A): The area bounded by $y^2=8x$ and $x^2=8y$ is 64/3 sq. units. Reason (B): The area bounded by $y^2=4ax$ and $x^2=4by$ is 16ab/3 sq. units. The correct answer is (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.	1
30.	The area of the circle $x^2 + y^2=16$ exterior to the parabola $y^2=6x$ is A) $4/3(4\pi-\sqrt{3})$ B) $4/3(4\pi+\sqrt{3})$ C) $4/3(8\pi-\sqrt{3})$ D) $4/3(8\pi+\sqrt{3})$	1
31.	Area (in square unit) lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the line x=0, x=2 is (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$	1
32.	Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y=3 is (in sq unit) (a) 2 (b) 9/4 (c) 9/5 (d) 9/2	1
33.	Area of the region bounded by the curve $y^2 = 4x$ and y = 2x is (a) 2/3 (b) 1/3 (c) 1/4 (d) 3/4	1
34.	The area enclosed between the curve $y^2 = x$ and $y = \|x\|$ is (a) 1/6 (b) 1/3 (c) 2/3 (d) 1	1
35.	The area enclosed between the curve $y = x$ *and* $y = 2x - x^2$ (in square units) is (a) 1/2 (b) 1/6 (c) 1/3 (d) 1/4	1
36.	The area enclosed between the curve $y = ax^2$ and $x = ay^2, (a > 0)$ is 1 square unit then the value of a is (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$	1
37.	The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x- axis and lying in the first quadrant is	1

	(a) 9 (b) 36 (c) 18 (d) 27/4	
38.	The area enclosed between the parabolas $y^2 = 4x$ and $x^2 = 4y$ is (in Square units) (a) 4/3 (b) 1/3 (c) 16/3 (d) 8/3	1
39.	Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is (a) $2(\pi - 2)$ (b) $\pi - 2$ (c) $2\pi - 1$ (d) $(d)2(\pi + 2)$	1
40.	The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$is equal to (a) $\pi^2 ab$ (b) πab (c) $\pi a^2 b$ (d) πab^2	1
41.	The area bounded by the parabola $y^2 = 8x$, the x-axis and the latus rectum is (a) 16/3 (b) 23/3 (c) 32/3 (d) 16√2/3	1
42.	If the area bounded by y-axis and curves y = cos x and y = sinx, $0 \le x \le b$ is (√2 -1) sq. units then the value of b is (a) π (b)π/2 (c) π/4 (d) none of these	1
43.	The area bounded by the curve $y^2 = x - 4$ and the lines $y = 0$ and $y = 5$ is (a) 38/3 (b) 76/3 (c) 19/3 (d) 57/3	1
44.	The area bounded by curve $y = \sin 2x$, x-axis and the lines $x = \pi/4$ and $x = 3\pi/4$ is: (a) 1 sq. units (b) 2 sq. units (c) 4 sq. units (d) 3 2sq. units	1
45.	Area under the curve $y = \sqrt{(b^2 - x^2)}$ included between the lines x = 0 and x = b is: (a) $\pi b^2 / 2$ (b) π b / 2 (c) π b / 4 (d) $\pi b^2 / 4$	1
46.	The area bounded by the curve $y = \tan^2 x$, x-axis and ordinates x =0 and x = π/4 is (a)π/4 (b) 1 + π/4 (c)1 – π/4 (d) none of these	1
47.	If area bounded by the curve $y(1 + 4x^2) = 1$, x-axis and ordinate x = 0 and x = a is π/8 sq. units, then the value of a is (a) 1/2 (b) 1 (c) -1/2 (d) none of these	1
48.	The area of the region bounded by the curve x = 2y + 3, y-axis and the line y = - 1 and y = b is 6 sq. units , then the value of b is (a)b= 0 (b) b= 1 (c) b= -1 (d) none of these	1
49.	The area of the region bounded by the curve $y = x^3$, the x-axis and the ordinates x = a, and x = 1 is 17/4 then value of a is	1

	(a) a = -2 (b) a=2 (c) a=1 (d) none of these	
50.	If the curve y = f(x) crosses x-axis into 3 times and areas A1,A2 and A3 are formed, then the area between the curve and the ordinates x = a and x = b is given by  (a) A1 – A2 +A3 (c) A1+A2 –A3 (b) A1 –A2 – A3 (d) A1+A2+A3	1
51.	The area bounded by the curve y = sin x, x = 0 and x= π is (a) 2 sq. unit (b) 4 sq. unit (c) 3 sq. unit (d) 1 sq. unit	1
52.	Area bounded by the curve y = f(x), x-axis and the lines x=a and x = b is: (a) $\int_a^b x\,dy$ (b) $\int_a^b y\,dx$ (c) $\int_a^b x^2\,dy$ (d) $\int_a^b y^2\,dx$	1
53.	The area bounded by the curve $y^2 = 4ax$ and its latus rectum is (a) $\frac{4}{3}a^2$ sq. units (b) $\frac{8}{3}a^2$ sq. units (c) $\frac{16}{3}a^2$ sq. units (d) None of these	1
54.	The area enclosed between y = x, x = 1, x = 3 and x-axis is (a) 2 sq. units (b) 9/2 sq. units (c) 4 sq. units (d) None of these	1
55.	The area between the curve $y = x^2$, x-axis and the lines x = 0 and x = 2 is (a) $\frac{2}{3}$ sq unit (b) $\frac{6}{3}$ sq unit (c) $\frac{8}{3}$ sq unit (d) $\frac{4}{3}$ sq unit	1
56.	The area of the region bounded by the curve $y^2 = x$ and the lines x = 1 and x = 4 is (in sq. units): (a) $\frac{15}{2}$ (b) $\frac{14}{3}$ (c) 7 (d) None of these	1
57.	The area enclosed between x-axis and the curve y = cos x when $0 \le x \le 2\pi$ is (a) 0 sq. unit (b) 2 sq. units (c) 3 sq. units (d) 4 sq. units	1
58.	Find the area of the region bounded by the curve $y = x^2$ and the line y = 16 is (a) $\frac{32}{3}$ (b) $\frac{256}{3}$ (c) $\frac{64}{3}$ (d) $\frac{128}{3}$	1
59.	The area bounded by the curve y = 4 sin x, x-axis from x = 0 to x = π is equal to: (a) 1 sq unit (b) 2 sq unit (c) 4 sq unit (d) 8 sq unit	1
60.	The area bounded by the parabola $y^2 = x$ and the straight line 2y = x is (a) $\frac{4}{3}$ sq. units (b) 1 sq. unit (c) $\frac{2}{3}$ sq. unit (d) $\frac{1}{3}$ sq. unit	1
61.	The area of the region bounded three roads and the equation of roads is given by the curve $y = x + 1$ and the line x=2 and x=3 is (a) $\frac{7}{2}$ *sq units* (b) $\frac{9}{2}$ *sq units* (c) $\frac{11}{2}$ *sq units* (d) $\frac{13}{2}$ *sq units*	1
62.	Using integration, find the area of cake which is cut in the shape of the quadrant of the circle	1

	of radius 2units and center (0,0). (a) 2π (b) 4π (c) 3π (d) π	
63.	The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is (a) $\frac{1}{3}$ $sq\ unit$ (b) $2\ sq\ unit$ (c) $\frac{4}{3}$ $sq\ unit$ (d) $\frac{2}{3}$ $sq\ unit$	1
64.	A Cable hangs in the form of parabola with its axis vertical. The cable is 10m high and 5m wide at the base (a) $y^2 = \frac{5}{8}x$ (b) $y^2 = -\frac{5}{8}x$ (c) $x^2 = \frac{5}{8}y$ (d) $x^2 = -\frac{5}{8}y$ 	1
65.	A parking lot in JNU CAMPAS has an area equals to the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line$x = \frac{a}{\sqrt{2}}$. This area is allotted for car owners who practices car pooling. On the basis of above information, find the area used for car pooling. (a) $\frac{a^2(\pi-2)}{2}$ $sq\ units$ (b) $\frac{a^2}{4}$ $sq\ units$ (c) $\frac{a^2(\pi-2)}{4}$ $sq\ units$ (d) $\frac{a^2(\pi-2)}{5}$ $sq\ units$	1
66.	The area bounded by the curve $y = \|x\|$, the x-axis and between $x = -2\ to\ x = 0$ is (a) $4\ sq\ units$ (b) $\frac{3}{2}$ $sq\ units$ (c) $1\ sq\ units$ (d) $2\ sq\ units$	1
67.	Ram and Aman both draw parabolas. Ram draw a parabola on positive y-axis whose equation is $y^2 = 4ax$ and Aman draw a parabola onpositive x-axis whose equation is $x^2 = 4ay$ on the same xy-plane, then her teacher told them to find the area bounded by these two parabolas. 	1

	` (a) $\frac{8a^2}{3}$ (b)$\frac{16a^2}{3}$ (c)$\frac{32a^2}{3}$ (d)$\frac{64a^2}{3}$	
68.	Mohit draw three lines and give the equation of lines as $3x - y - 3 = 0, 2x + y - 12$ $and\ x - 2y - 1 = 0$ $and\ told\ his\ brother\ to\ find\ the\ area\ bounded\ by\ these\ lines$  $(a)8\ sq.units$ (b) $9\ sq.units$ (c) $10\ sq.units$ (d) $11\ sq.units$	1
69.	The area of region bounded by the line $2x + y = 8$, the Y-axis and the lines y=2 and y=4 is $(a)5\ sq.units$ (b) $6\ sq.units$ (c) $12\ sq.units$ (d) $7\ sq.units$	1
70.	The area bonded by the parabola$y^2 = 16x$ and its latusrectum is (a) $\frac{25}{3}\ sq.units$ (b)$\frac{16}{3}\ sq.units$ (c) $\frac{64}{3}\ sq.units$ (d)$\frac{32}{3}\ sq.units$	1

ANSWERS:

Q. NO	ANSWER	MARKS
1.	a) 2 sq unit	1
2.	e) 4/3 sq unit	1
3.	a) $A^{2}/6$ sq unit	1
4.	b) 6 sq unit	1
5.	c) $\frac{17\sqrt{17}}{6}$ sq unit	1
6.	d) 1 sq unit	1
7.	b) 9sq. unit	1
8.	c) 9sq. unit	1
9.	c) 12 sq. unit	1
10.	b) $\frac{3}{2}$sq. unit	1
11.	b	1
12.	a	1
13.	b	1
14.	b	1
15.	b	1
16.	b	1
17.	b	1
18.	a	1
19.	b	1
20.	a	1
21.	c	1
22.	d	1
23.	b	1
24.	a	1
25.	b	1
26.	b	1
27.	a	1
28.	a	1
29.	a	1
30.	c	1
31.	a	1
32.	b	1
33.	b	1
34.	b	1
35.	b	1
36.	a	1

37.	a	1
38.	a	1
39.	b	1
40.	b	1
41.	Option – c	1
42.	Option – b	1
43.	Option – b	1
44.	Option – a	1
45.	Option – d	1
46.	Option – c	1
47.	Option – a	1
48.	Option – b	1
49.	Option – a	1
50.	Option – d	1
51.	a	1
52.	c	1
53.	b	1
54.	c	1
55.	c	1
56.	b	1
57.	d	1
58.	b	1
59.	d	1
60.	a	1
61.	(a) $\frac{7}{2}$ $sq\ units$	1
62.	(d) π	1
63.	(c)$\frac{4}{3}$ $sq\ unit$	1
64.	(c)$x^2 = \frac{5}{8}y$	1
65.	(c) $\frac{a^2(\pi-2)}{4}$ $sq\ units$	1
66.	(d) 2 $sq\ units$	1
67.	(b)$\frac{16a^2}{3}$	1
68.	(d) 11 $sq.units$	1
69.	(a) 5 $sq.units$	1
70.	(c)$\frac{64}{3}$ $sq.units$	1

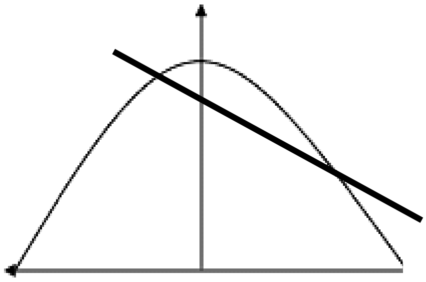
CHAPTER-8

APPLICATION OF INTEGRALS

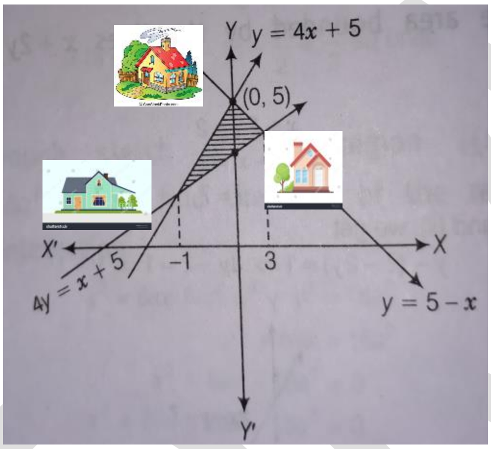
02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the area bounded by the curve y= x |x|, x-axis and x= -1 and x=1.	2
2.	Find the area bounded by the lines |x|+|y|= 1.	2
3.	Find the area bounded by the curves $y = x^2$ and the line y=4.	2
4.	Find the area of the curve $y = sinx$ between 0 and π.	2
5.	Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the $x - axis$ in the first quadrant.	2
6.	Find the area between the curves $y = x$and $y = x^2$.	2
7.	Write the formula of $\int \sqrt{a^2 - x^2}\, dx$	2
8.	Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.	2
9.	Write the Geometric significance of the integral $\int_a^b f(x)$dx.	2
10.	Using integration, Find the area of the region bounded by the line 2y= -x+8, X-axis and the lines X=2 andx=4.	2
11.	Find the area bounded by the curvey^2= 2y-xand Y axis.	2
12.	Find the area of the region bounded by the curve x^2=4y and the straight line x=4y-2.	2
13.	Find the area of the region bounded by the curve X axis and y = 2x-x^2.	2
14.	Using integration find the area of the region bounded by the line 2y = -x+8 , x-axis and the line x = 2 and x = 4.	2
15.	Using integration find the area of the region bounded between the line x = 4 and the parabola $y^2 = 4x$.	2
16.	Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	2
17.	Find the area of the region bounded by the curve $y^2 = x$ and the line x = 1, x = 4 and the x- axis.	2
18.	Find the area of the region bounded by the curve parabola $y = x^2$ *and the line* $y = \|x\|$.	2
19.	Find the area bounded between y = $\sin^{-1}$x and y-axis between y = 0 and y = π/2.	2
20.	If the area bounded by the curve y = 3x, x-axis and between the ordinates x = 1 and x = b is 12 sq..units, then find the value of b.	2
21.	If the area bounded by the parabola y^2 = 16x and the line x=a is 128/3 sq. units, then find the value of a.	2
22.	Using integration check whether given statement is true or false Statement: The region under the curve y = √(1 – x^2) on the interval [-1,1] has area A = π/2,	2
23.	Find the area of the region bounded by the $y = \|x - 5\|$ and ordinates x=0 and x=1.	2
24.	Using integration, find the area of the region bounded by: y=mx (m > 0, x= 1, x= 2 and the x-axis).	2
25.	Sketch the region bounded by the lines 2x+y = 8, y = 2, y =4 and the y-axis. Hence, obtain its area, using integration.	2
26.	Find the area bounded by y = x^2, the x-axis and the lines x = 1 and x = -1.	2
27.	Find the area bounded by the curve y = x^3 , x = -2 and x = 1.	2

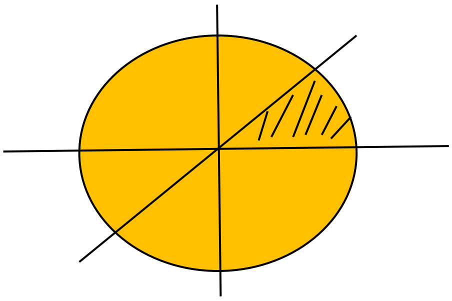
<table>
<tr><td>28.</td><td>Find the area of the region bounded by the parabola $y^2 = 8x$ and the line x = 2.</td><td>2</td></tr>
<tr><td>29.</td><td>Reshma draw a beautiful painting in which she draw mountains, trees, birds, river, houses etc. His little brother come across the painting and cut one of the mountain by drawing a straight line. Based on the above information find the area bounded by mountain and straight line . The equation of mountain is $y = -x^2$ and equation of straight line is $x + y + 2 = 0$


</td><td>2</td></tr>
<tr><td>30.</td><td>Find the area bounded by the curve $y^2 = 9x \ and \ y = 3x.$</td><td>2</td></tr>
<tr><td>31.</td><td>Location of the three houses of a society is represented by points A(0,5), B(3,2) and C (1,1). Find the area bounded by these three houses and the equqtion of line represented by house A, B, C are $y = 4x + 5,$

$$y = 5 - x, and \ 4y = x + 5.$$



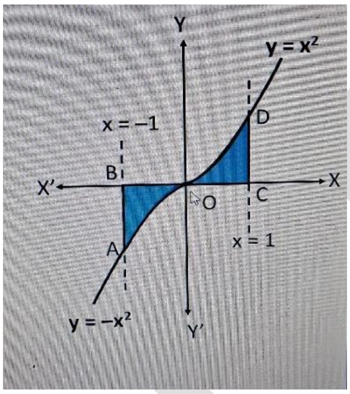
</td><td>2</td></tr>
<tr><td>32.</td><td>A circular Pizza is cut into 8 equal pieces with the help of knife then find the area of region bounded by each pieces of pizza if the equation of pizza and knife is represented by $x^2 + y^2 = 32 \ and \ y = x$ respectively.


</td><td>2</td></tr>
<tr><td>33.</td><td>Consider the following curve and find the area under the curve $y = 2\sqrt{x}$ included between the line x=0 and x=1 is</td><td>2</td></tr>
</table>

ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>
We know Y= x |x|

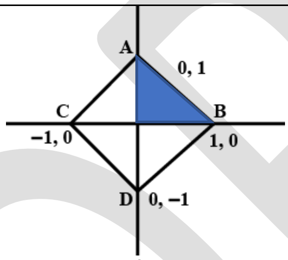
Y= $\{x^2 \quad if\ x > 0 \ -x^2 \quad if\ x < 0$



Area required = Area ABO + Area DCO

Area ABO = $\int_{-1}^{0} y$ dx = $\int_{-1}^{0} -x^2$ dx = -(1/3) since area is always positive so area ABO is 1/3

Area DCO = $\int_{-1}^{0} y$ dx = $\int_{-1}^{0} .x^2$ dx = (1/3)

So, required area is 1/3 + 1/3 = 2/3.
</td><td>2</td></tr>
<tr><td>2.</td><td>


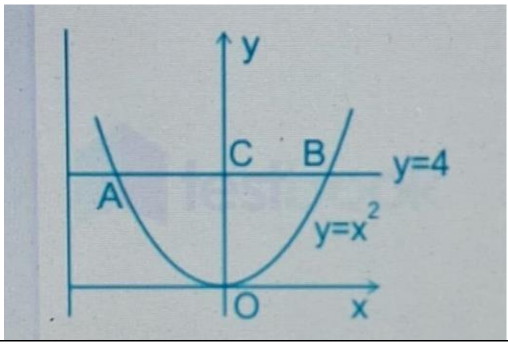
Area ABO = $\int_{-1}^{0} y$ dx where the shaded part having the oblique line equation be x + y = 1 so, y = 1 – x

Therefore Area ABO = $\int_{-1}^{0} (1 - x)$ dx

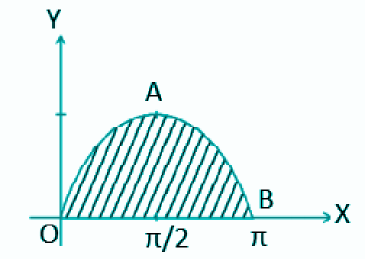
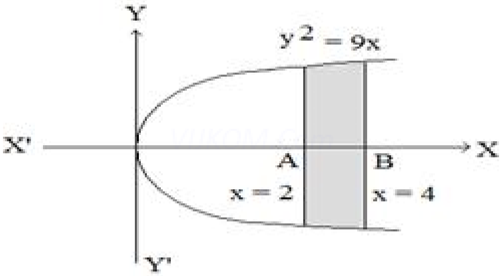
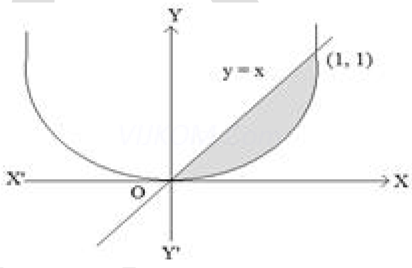
= 1/2

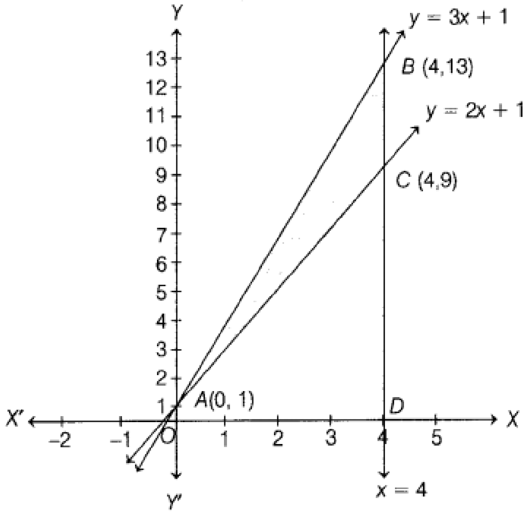
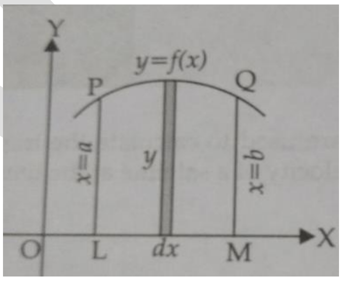
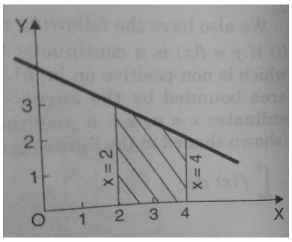
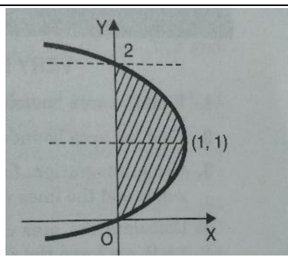
So, required area is 4 * Area of AOB = 4*(1/2) = 2sq. unit
</td><td>2</td></tr>
<tr><td>3.</td><td>
We have $y = x^2$ and y = 4

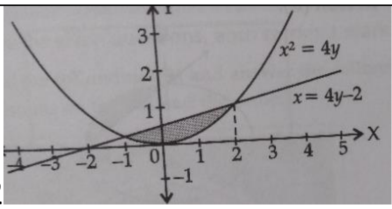
Let AB represent the line y=4

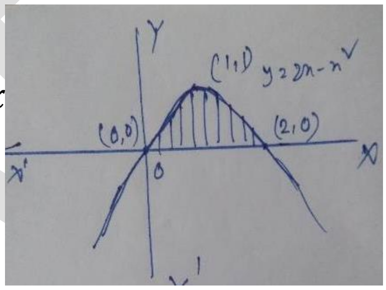
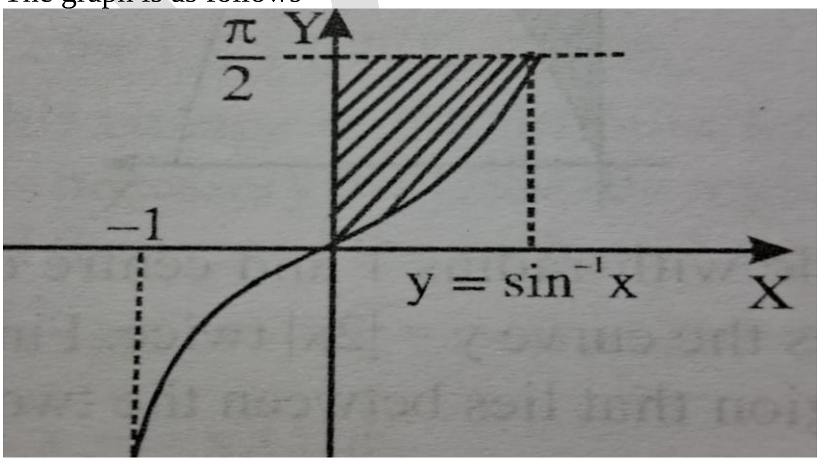


</td><td>2</td></tr>
</table>

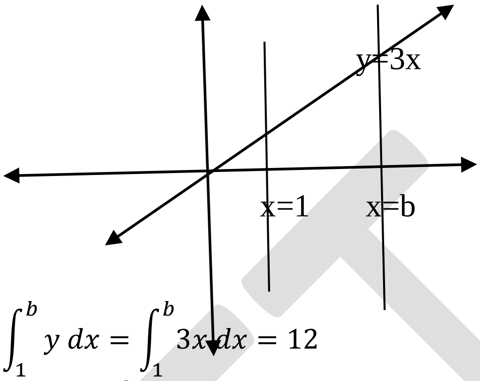
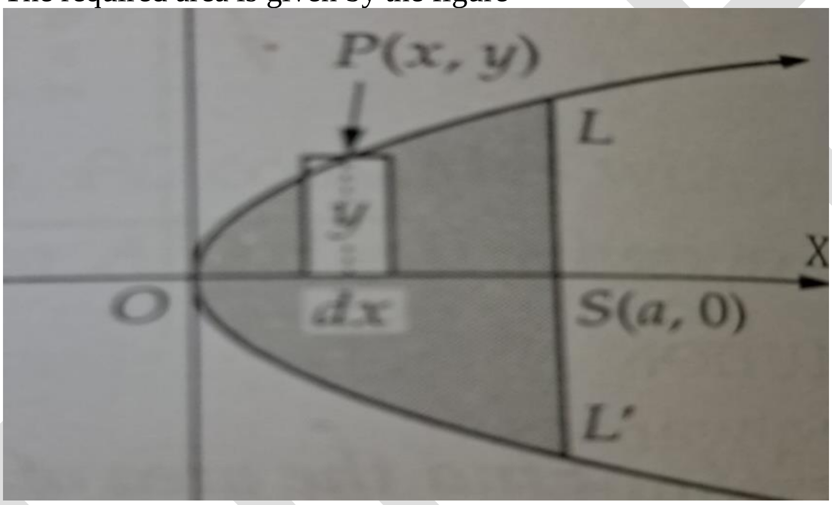
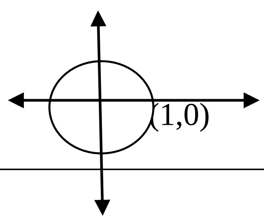
	Let AOB represent $y = x^2$ i.e $x = \pm\sqrt{y}$ Since BOCB is in the 1st quadrant , we use only positive value of $\sqrt{y}$ Area of AOBA= $2 * \int_0^4 \sqrt{y}\, dy$ = (32/3) sq. unit	
4.	 $y = sinx$ Area of OAB=$\int_0^{\pi} ydx$ =$\int_0^{\pi} sindx$ = 2 sq. units	2
5.	$y^2 = 9x$, $x = 2$, $x = 4$ and the $x - axis$ in the first quadrant  Required area=$\int_2^4 ydx$ =$\int_2^4 \sqrt{9x}dx$ =$\int_2^4 3\sqrt{x}dx$ =16-4$\sqrt{2}$ sq. units	2
6.	$y = x$ $y = x^2$ On solving x = 0, 1 $Area = \int_0^1 (x - x^2)\, dx$ $= \frac{1}{6} sq\ unit.$ 	2
7.	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$	2
8.	Given eq. of the lines are $y = 2x + 1$ -------(1) $y = 3x + 1$ ------(2) $x = 4$ -----(3)	2

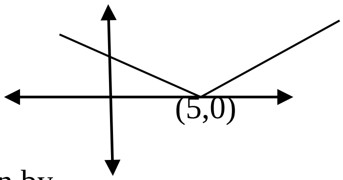
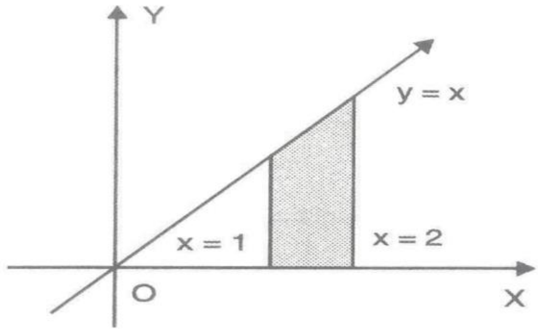
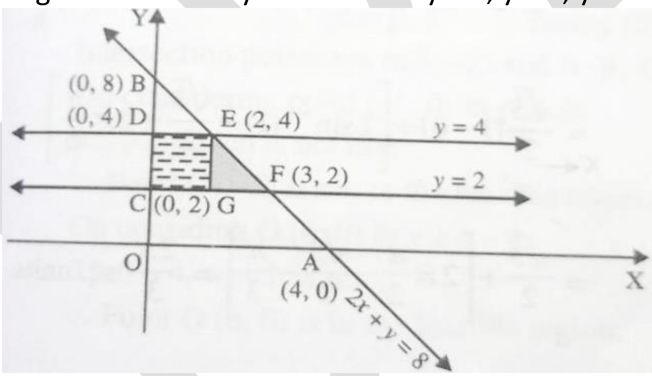
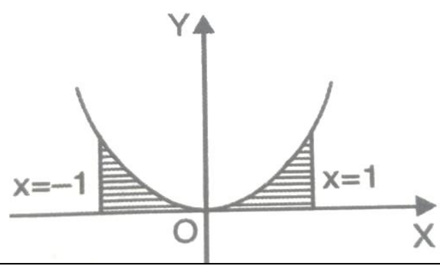
	 By solving these equations we get the vertices of triangle as A(0, 1), B(4, 13) and C(4, 9). ∴ Required area = Area (OABDO) – area (OACDO) $=\int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$ = 8sq. units	
9.	Let there be an arbitrary strip of height y and width dx. Area of elementary strip dA = ydx,where y=f(x).total are A of the region between X-axis ordinates x=a, x=b and the curve y=f(x) Sum of the areas of elementary strips across PQML A=$\int_a^b ydx = \int_a^b f(X)$ dx  	2
10.	We have 2y=-x+4 ⇨ x+2y=4 ⇨ x/4+y/2=1 Required area is $\int_2^4 y$dx=$\int_2^4 (-\frac{x}{2}+4)$dx =$(-\frac{x^2}{4}+4x)\vert_2^4$=5 sq. units  	2
11.	We have y^2=2y-x (y-1)^2=-(x-1) When x=0 then y=0,2 Required area is =$\int_0^2 x$dy=$\int_0^2 (2y-y^2)$ dy =$(y^2-y^3/3)\ \vert_0^2$=4/3 sq. units.  	2
12.	We have x^2=4y and the line x=4y-2	2

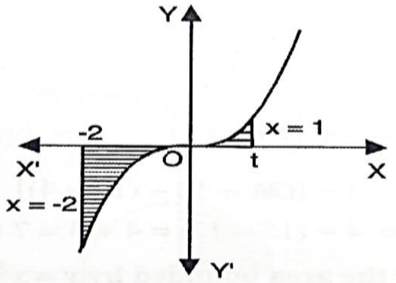
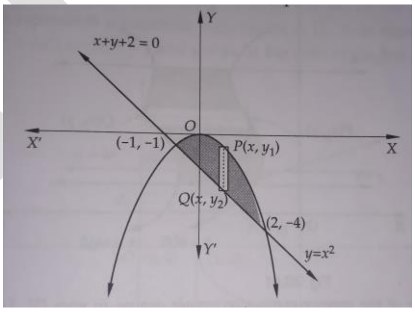
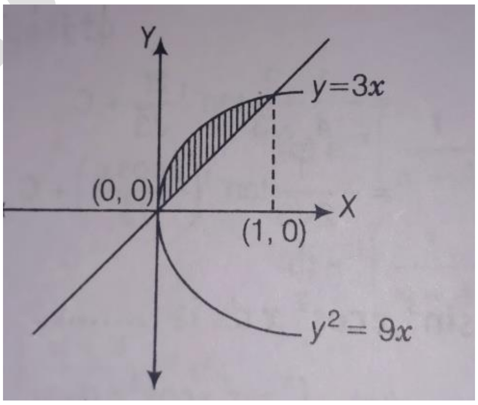
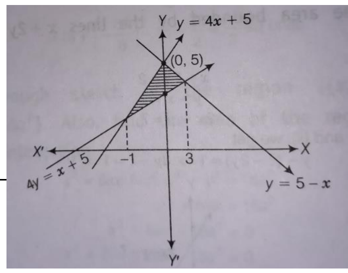
<table>
<tr><td></td><td>

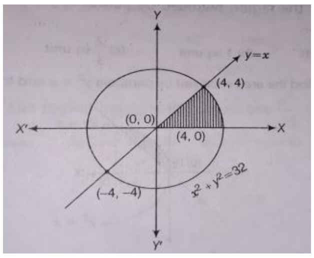
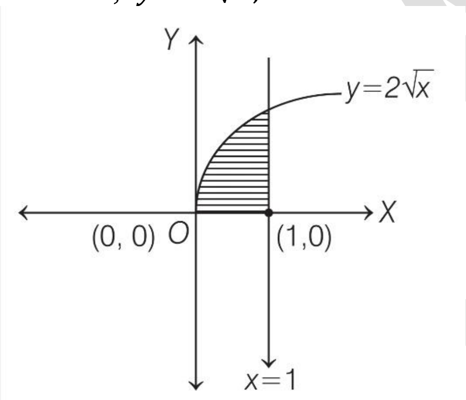
Then x=4y-2
⇨x-4y=-2
⇨x/(-2)+y/(1/2)=1
$$x^2 = x + 2$$
X=-1, 2
The parabola and the line intersect at the point (-1,1/4) and (2,1)
∴ The required area is $=\int_{-1}^{2} y_1 dx - \int_{-1}^{2} y_2 dx$
$=\int_{-1}^{2}(\frac{x+2}{2} - x^2/4)dx$
$=\frac{1}{4}(\frac{x^2}{2} + 2x - \frac{x^3}{3}) |_{-1}^{2}$
$=\frac{9}{8}$ sq.units</td><td></td></tr>
<tr><td>13.</td><td>We have y=2x-x^2
⇨$(x-1)^2$=-(y-1)
∴ the required area is $=\int_0^2 y dx$
$=\int_0^2 (2x - x^2)\, dx$
$=(2\frac{x^2}{2} - \frac{x^3}{3})|_0^2$
$=\frac{4}{3}$sq. units

</td><td>2</td></tr>
<tr><td>14.</td><td>A=$\int_2^4 y dx = \int_2^4 \left(\frac{-x+8}{2}\right) dx = 5 square\ unit$</td><td>2</td></tr>
<tr><td>15.</td><td>A=$2\int_0^4 y dx = 2\int_0^4 \sqrt{16x}\ dx = 8\int_0^4 \sqrt{x}\, dx = \frac{128}{3}$square units</td><td>2</td></tr>
<tr><td>16.</td><td>A= $4\int_0^a y dx = 4\int_0^a b/a\sqrt{a^2 - x^2} dx = \pi ab$</td><td>2</td></tr>
<tr><td>17.</td><td>A= $\int_1^4 y dx = \int_1^4 \sqrt{x} dx = 14/3\ square\ units$</td><td>2</td></tr>
<tr><td>18.</td><td>A2 $\int_0^1 (x - x^2)dx - \frac{1}{3}\ square\ units$</td><td>2</td></tr>
<tr><td>19.</td><td>The graph is as follows


The required area is given by</td><td>2</td></tr>
</table>

	$$= \int_0^{\pi/2} siny\, dy = [-cosy]_0^{\frac{\pi}{2}} = 1$$	
20.	The graph is as follows  The required area is given by $$= \int_1^b y\, dx = \int_1^b 3x\, dx = 12$$ $$\frac{3b^2}{2} = \frac{27}{2}$$ b = 3	2
21.	The required area is given by the figure  So, The required area is given by $$A = 2\int_0^a y\, dx = 2\int_0^a \sqrt{16x}\, dx$$ $$\frac{128}{3} = 8\int_0^a x^{1/2}\, dx = \frac{16}{3}\, a^{3/2}$$ a = 4	2
22.	The region is given by the figure  	2

	So, the area is $$A = 4\int_0^1 y\,dx = 4\int_0^1 \sqrt{(1-x^2)}\,dx$$ $$A = \pi$$ (False)	
23.	The area is given by figure as follows  The required area is given by $$= \int_0^1 y\,dx = \int_0^1 \lvert x-5\rvert\,dx = \frac{9}{2} sq.units$$	2
24.	Required area = shaded area  $=\int_1^2 mx\,dx$ $=\frac{3}{2}$ m sq. unit	2
25.	Region bounded by the lines 2x+y = 8, y = 2, y=4 is as shown in the below figure:  Reqd. area = ar(CDEG) + ar(ΔGEF) = 2×2 + $\int_2^3 (8-2x)dx$ = 7 sq units	2
26.	 Reqd. area = $\int_{-1}^{1} x^2\,dx$ $=\frac{2}{3}$ sq. units	2
27.	We have y = x^3	2

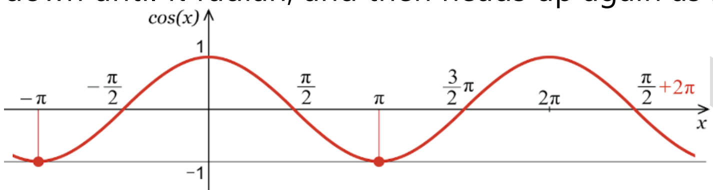
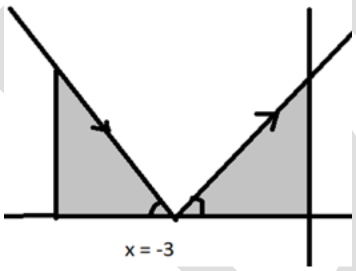
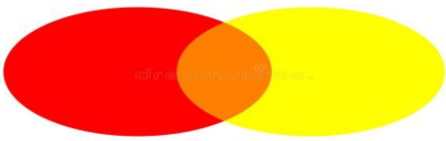
	$\therefore$ **Reqd. area** $= \left\| \int_{-2}^{0} x^3\,dx \right\| + \int_{0}^{1} x^3\,dx$  $= \left\| \left[\frac{x^4}{4}\right]_{-2}^{0} \right\| + \left[\frac{x^4}{4}\right]_{0}^{1}$ $= \left\| \left(0 - \frac{16}{4}\right) \right\| + \left(\frac{1}{4} - 0\right) = \frac{16}{4} + \frac{1}{4} = \frac{17}{4}.$	
28.	Reqd. area = $2\int_0^2 \sqrt{8x}\,dx$ $= \frac{8}{3}\sqrt{2}\,[2^{\frac{3}{2}} - 0]$ = 32/3 sq. units	2
29.	Required area = $\left(\int_{-1}^{2}(y_1 - y_2)dx\right)$ $= \int_{-1}^{2} -x^2 - x - 2\,dx$ $= \int_{-1}^{2} -x^2 + x + 2\,dx$ $= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$ $= \left(-\frac{8}{3} + 6\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$ $= \frac{9}{2}\,sq.units$  	2
30.	We have $y^2 = 9x$ and $y = 3x$ $. \Rightarrow (3x)^2 = 9x$ $. \Rightarrow 9x^2 = 9x$ $. \Rightarrow 9x(x-1) = 0$ $. \Rightarrow x = 0,1$ $\therefore$ Required bounded area $= \int_0^1 \sqrt{9x}dx - \int_0^1 3x dx$ $= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$ $= 3\left(\frac{2}{3} - 0\right) - 3\left(\frac{1}{2} - 0\right)$ $= 2 - \frac{3}{2}$ $= \frac{1}{2}\,sq\ units$  	2
31.	$\therefore$ Required bounded area between three houses $= \int_{-1}^{0}(4x+5)dx - \int_0^3 (5-x)dx - \frac{1}{4}\int_{-1}^{3}(x+5)dx$ $= \left[\frac{4x^2}{2} + 5\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}[x^2 + 5x]_{-1}^{3}$ $= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$  	2

	$=3+\frac{21}{2}-\frac{1}{4}.24$ $=-3+\frac{21}{2}=\frac{15}{2}\ sq\ units$	
32.	∴ Required area of each slice of pizza  $= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 - x^2}\, dx$ $= \left\|\frac{x^2}{2}\right\|_0^4 + \left\|\frac{x}{2}\sqrt{\left(4\sqrt{2}\right)^2 - x^2} + \frac{4\sqrt{2}}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right\|_4^{4\sqrt{2}}$ $=\frac{16}{2} + \left[\frac{4\sqrt{2}}{2}.0 + 16\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}} - \frac{4}{2}\sqrt{\left(4\sqrt{2}\right)^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$ $=8+\left[16.\frac{\pi}{2} - 2.\sqrt{16} - 16.\frac{\pi}{4}\right]$ $=8+[8\pi - 8 - 4\pi]$ $=4\pi\ sq\ units$	2
33.	We have, $y = 2\sqrt{x},\ x = 0\ and\ x = 1$  ∴ Area of shaded region = $\int_0^1 2\sqrt{x}\, dx$ $=2\left[\frac{x^{\frac{3}{2}}}{3}.2\right]_0^1 = 2\left(\frac{2}{3}.1 - 0\right) = \frac{4}{3} sq\ units$	2

CHAPTER-8

APPLICATION OF INTEGRALS

03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the area of Δ ABC, the coordinates of whose vertices are A (2, 5), B(4, 7) and C(6, 2) by using integration.	3
2.	If y= 2 sin x + sin 2x for $0 \leq x \leq 2\pi$ find the area enclosed by the curve and the x-axis.	3
3.	Find the area of the region bounded by the ellipse $\frac{y2}{16} + \frac{x2}{25}$ =1.	3
4.	Find the area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and $x - axis$.	3
5.	Find the area of the region bounded by the curve $y = x^2$ and $y = 16$.	3
6.	Find the area under the curve $y = x^2$ and the lines $x = -1, x = 2$ and $x - axis$	3
7.	Find the area bounded by the curve y = cos x, x-axis and the ordinates x = -5π/6 and x = π	3
8.	Find the area of larger portion of the circle $x^2 + y^2 = 4$ cut off by the line x=1	3
9.	If the area of the region enclosed by the parabola $y^2 = 4ax$ and the line y = mx is 3/8, then find a relation between a and m.	3
10.	In a classroom, the teacher explains the properties of a particular curve by saying that this particular curve has beautiful ups and downs. It starts at 1 and heads down until π radian, and then heads up again as shown in the figure  Then find the area enclosed by the curve, x = -π and x = π.	3
11.	A ray is reflected according to the below given diagram.  If both the mentioned angles and shaded regions are equal then find the graph of the curve and area of the shaded region.	3
12.	Find the area enclosed by the circle $x^2 + y^2 = 2$.	3
13.	Rishika made two chapattis and place one upon the other as shown in the figure. One of the chapatti representes the equation $(x-2)^2 + y^2 = 4$, while other chapatti represents the equation $x^2 + y^2 = 4$ 	3

Based on the above information, answer the following questions.

(i) Find the centre and of the circle of equation $(x-2)^2 + y^2 = 4$,

(a) C=(2,0), r =2 (b) C=(0,0),r =2

(b) C=(2,0) , r=1 (d) C= (0.2), r=2

(ii) Both the chapattis meet each other at

(a) $(1,\sqrt{3}),(1,-\sqrt{3})$

(b) $(1,\sqrt{3}),(1,-3)$

(c) $(1,3),(1,-3)$

(d) $(1,\sqrt{2}),(1,-\sqrt{2})$

(iii) Area bounded by two chapattis is

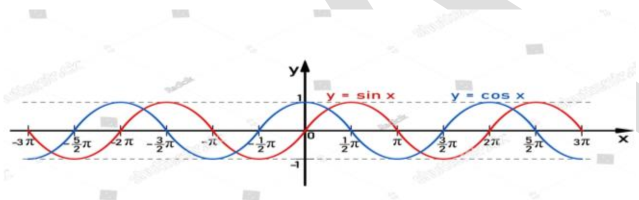
(a)$\frac{8\pi}{3}-\sqrt{3}\ \ sq.units$

(b)$\frac{8\pi}{5}-2\sqrt{3}\ \ sq.units$

(c)$\frac{8\pi}{3}-2\ \ sq.units$

(d)$\frac{8\pi}{3}-2\sqrt{3}\ \ sq.units$

14. In a classroom teacher explain the properties of a particular curve by saying that this particular curve has beautiful up and downs. It starts at 1 and heads down until $\pi\ radian$, and then heads up again and closely related to sine function and both follow each, other exactly $\frac{\pi}{2}\ radian$ apart as shown in figure. 3



Based on the above information ,answer the following questions.

(i) Name the curve, about which teacher explained in the classroom.

(a) cosine (b) sine

(c) tangent (d) cotangent

(ii)Area of curve explained in the passage from 0 to$\frac{\pi}{2}$ is

(a) $\frac{1}{3}\ sq\ units$

(b) $\frac{1}{2}\ sq\ units$

(c) $1\ \ sq\ units$

(d) $2\ sq\ units$

(iii) Area of curve discussed in classroom from $\frac{\pi}{2}\ to\ \frac{3\pi}{2}$ is

(a)$\frac{7}{2}\ sq\ units$

(b) $\frac{9}{2}\ sq\ units$

(c) $\frac{11}{2}\ sq\ units$

(d) $\frac{13}{2}\ sq\ units$

15. In geometry we have learn formulae to calculate areas of various geometrical figures including triangles,rectangles, trapezium and circle. Such formula is fundamental in the application of Mathematics to many real-life problems.The formula of geometry allow us to calculate area of many simple figure .However, they are inadequate for calculating the areas 3

enclosed by curves. For that we need concept of integral calculus.

(i) The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ is$

(a)$\pi b\ sq.units$
(b) $\pi a\ sq.units$
(c) $\pi\ sq.units$
(d) $\pi ab\ sq.units$

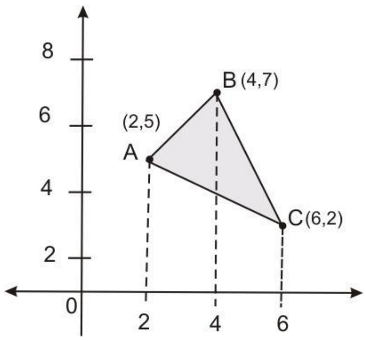
(ii) The area enclosed by the circle $x^2+y^2=a^2\ is$

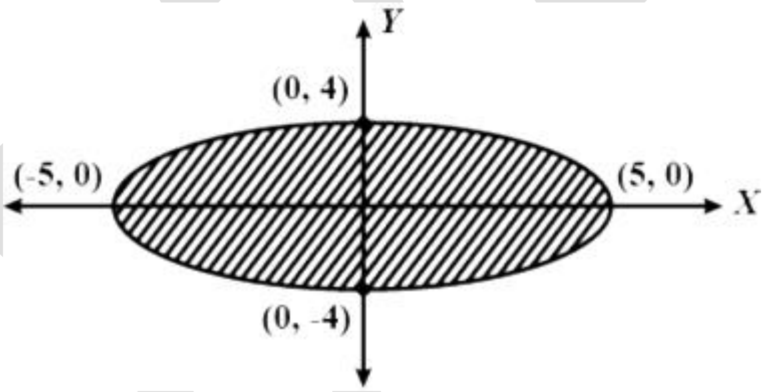
(a)πa^2 (b) π (c) a^2 (d) a

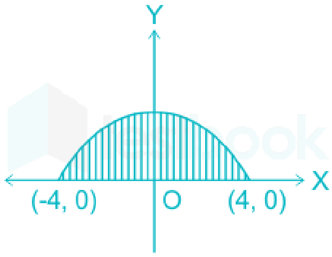
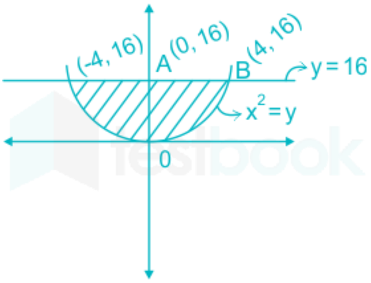
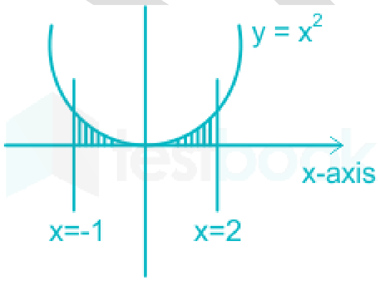
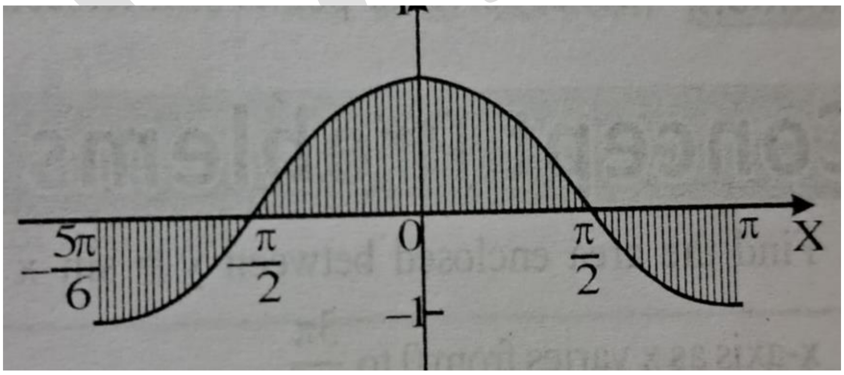
(iii) The area of the region bounded by the curve $y=x^2\ and\ the\ line\ y=4$

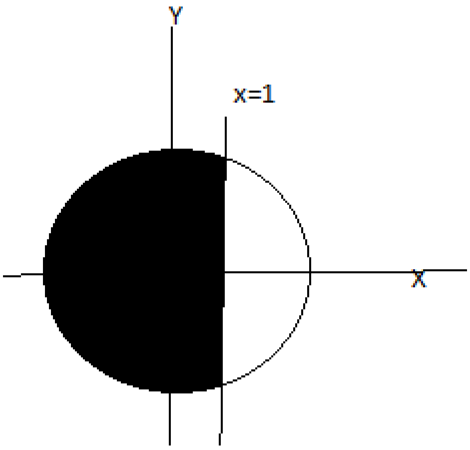
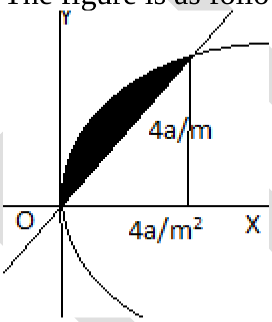
(a) 32 (b) 32/3 (c) 3 (d) 23

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Vertices of the given triangle are A(2,5) B(4,7) and C(6,2) Equation of AB $y-5 = \frac{7-5}{4-2}(x-2)$ ⇨ $y - 5 = x-2$ ⇨ $y = x+3$  The equation of side BC, $$(y-7) = \frac{2-7}{6-4}(x-4)$$ $$(y-7) = \frac{-5}{2}(x-4)$$ $$2y - 14 = -5x + 20$$ $$2y = -5x + 34$$ $$y = \frac{1}{2}(-5x + 34) \qquad -(2)$$ The equation of side AC, $$(y-5) = \frac{2-5}{6-2}(x-2)$$ $$(y-5) = \frac{-3}{4}(x-2)$$ $$4y - 20 = -3x + 6$$ $$4y = -3x + 26$$	

	$$y = \frac{1}{4}(-3x + 26) \qquad -(3)$$ $\therefore$ Area of ΔABC $$= \int_2^4 y_{AB}\, dx + \int_4^6 y_{BC}\, dx - \int_2^6 y_{AC}\, dx$$ $$= \int_2^4 (x+3)\, dx + \int_4^6 \frac{-1}{2}(5x - 34)\, dx - \int_2^6 \frac{-1}{4}(3x - 26)\, dx$$ $$= 12 + \frac{1}{2}(18) - \frac{1}{4}(56) - 12 + 9 - 14 = 7 \text{ sq units}$$	
2.	To find the area enclosed by the curve and the x-axis, we need to integrate the absolute value of the function y with respect to x, between the limits 0 and 2π. The function y = 2 sin x + sin 2x is always non-negative for 0 ≤ x ≤ 2π, so we can simply integrate it as is. A=2 (2sin x + sin 2x)dx=2∫(2sin x + sin 2x) dx $=4 \int_0^{\pi} \sin x$ dx+$2\int_{\pi}^{2\pi} \sin 2x$ dx= 8 + 0 = 8	
3.	 Given the equation of the ellipse is $\frac{y2}{16} + \frac{x2}{25}$ =1 ⇨ $\frac{y2}{16}$ =1 - $\frac{x2}{25}$ ⇨ Y = $\frac{4}{5}\sqrt{25 + x^2}$ Since ellipse is symmetrical about the axes, So, required area = 4* $\int_0^5 (4/5)\sqrt{25 - x^2}$ dx = 20 π sq. units	

4.	$y = \sqrt{16 - x^2}$ At x-axis y will be 0 $0 = \sqrt{16 - x^2}$ $x = \pm 4$  Area of the curve $= \int_{-4}^{4} y dx$ $= \int_{-4}^{4} \sqrt{16 - x^2} dx$ $= 8\pi$ sq. unit	3
5.	Given equation of the curve are $y = x^2$ -------(1) $y = 16$ -------(2) From (1) and (2) $x = \pm 4$  Required area$= \int_{-4}^{4} y dx$ $= \int_{-4}^{4} (16 - x^2) dx$ $= 2\int_{0}^{4} (16 - x^2) dx$ $= \frac{256}{3}$ sq. units	3
6.	Given equation of the curve are $y = x^2$ -------(1) $x = -1$ -------(2) $x = 2$ --------(3)  Required area$= \int_{-1}^{2} y dx$ $= \int_{-1}^{2} x^2 dx$ =3 sq. units	3
7.	The graph of the function is as follows  Solving equation cos x =0 between [-5π/6 ,π] we get that the graph of function intersect x-axis at two points x = -π/2 and x = π/2 so, the required area is given by $$= \int_{-5\pi/6}^{\pi} \lvert cosx \rvert dx$$	3

	$= -\int_{-5\pi/6}^{-\pi/2} cosx\, dx + \int_{-\pi/2}^{\pi/2} cosx\, dx + \int_{\pi/2}^{\pi} cosx\, dx = 7/2$	
8.	The graph of the function cut off by line is as follows  As per figure the area of small portion is given by =area ABCA $= 2\int_1^2 y\, dx = 2\int_1^2 \sqrt{(4 - x^2)}\, dx$ $= 2[\frac{x\sqrt{2^2 - x^2}}{2} + \frac{2^2}{2} sin^{-1}\frac{x}{2}]_1^2$ $= \frac{4\pi - 3\sqrt{3}}{3}$ So Required area is $= \pi(1)^2 - \frac{4\pi - 3\sqrt{3}}{3} = \frac{3\sqrt{3} - \pi}{3}$	3
9.	The figure is as follows  Solving y^2=4ax and y = m x gives point of intersection ($4a/m^2$, 4a/m) $A = \int_0^{4a/m^2}(2\sqrt{ax} - mx)dx = [\frac{4}{3}\sqrt{ax^{\frac{3}{2}}} - \frac{mx^2}{2}]_0^{4a/m^2}$ $\frac{3}{8} = \frac{8a^2}{3m^3}$ $m^3 = a^2$	3
10.	Reqd. area = $4\int_0^{\pi/2} \cos x\, dx$ = 4 $[\sin x]_0^{\pi/2}$ =4 × 1 = 4 sq. units	3
11.	Reqd. area = $\int_{-6}^{0} \lvert x + 3 \rvert dx$ =$\int_{-6}^{-3} \lvert x + 3 \rvert dx$ + $\int_{-3}^{0} \lvert x + 3 \rvert dx$ = 2 $\int_{-3}^{0} (x + 3) dx$ = 9 sq. units	3

12.	Reqd. area = $4\int_0^{\sqrt{2}} \sqrt{2-x^2}\ dx$ $= 2\pi$ sq. units	3
13.	(i) (a) Given eq. of circle is $(x-2)^2 + y^2 = 4,$ $\Rightarrow (x-2)^2 + (y-0)^2 = 2^2,$ Eq. of circle $(x-h)^2 + (y-k)^2 = r^2, where\ centre\ (h,k)\ and\ radius = r$ So, by comparing above eq. we get centre (2,0) and radius = 2 (ii) (a)$(x-2)^2 + y^2 = 4 \ldots\ldots..(1)$ $x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2$…….(2) From eq.(1) and (2) we get $(x-2)^2 + 4 - x^2 = 4$ $x^2 - 4x + 4 + 4 - x^2 = 4$ $-4x + 4 = 0 \Rightarrow x = 1$ On putting x=1 in $x^2 + y^2 = 4 \Rightarrow 1^2 + y^2 = 4 \Rightarrow y^2 = 3 \Rightarrow y = \pm\sqrt{3}$ Therefore point of intersections are $\left(1,\sqrt{3}\right), (1,-\sqrt{3})$ (iii) (d) Required area =$2\left(\int_0^1 y_1 dx + \int_1^2 y_2\ dx\right)$ $= 2\left(\int_0^1 \sqrt{4-x^2}dx + \int_0^1 \sqrt{4-(x-2)^2}dx\right)$ $= \left[x\sqrt{4-(x)^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2 + \left[(x-2)\sqrt{4-(x-2)^2} + 4\sin^{-1}\frac{x-2}{2}\right]_0^1$ $= 4\sin^{-1}1 - \left(\sqrt{3} + 4\times\frac{\pi}{6}\right) + \left\{-\sqrt{3} + 4\sin^{-1}\left(-\frac{1}{2}\right)\right\} - \{0 +$ $4\sin^{-1}(-1)\}$ $=4\times\frac{\pi}{2} - \left(\sqrt{3} + \frac{2\pi}{3}\right) + \left(-\sqrt{3} - \frac{4\pi}{6}\right) - \left(-\frac{4\pi}{3}\right)$ $=\frac{8\pi}{3} - 2\sqrt{3}\ sq.units$	3
14.	(i) (a) Here the teacher explained about cosine curve. (ii) (c) ∴ Required area =$\int_0^{\frac{\pi}{2}} cosxdx$ $=[sinx]_0^{\frac{\pi}{2}}$ $= sin\frac{\pi}{2} - \sin 0$ $= 1 - 0 = 1\ sq\ units$ (iii) (b) ∴ Required area =$\left\lvert\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} cosxdx\right\rvert$ $= \left\lvert[sinx]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right\rvert$ $= \left\lvert sin\frac{3\pi}{2} - sin\frac{\pi}{2}\right\rvert$ $= \lvert -1 - 1\rvert$ $= \lvert -2\rvert = 2\ sq\ units$	3
15.	(i) (d) The given equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ldots\ldots\ldots\ldots\ldots (1)$ Area of ellipse = 4(area of region 1st quadrant) $=4\int_0^a y\ dx$ $= \int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$ $\left[\because (1) \Rightarrow y = \pm\frac{b}{a}\sqrt{a^2-x^2}\right]$	3

(But region OABO lies in 1st quadrant , y is positive)

$=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$

$=\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$=\frac{4b}{a}\left[\left\{\frac{a}{2}(0)+\frac{a^2}{2}\sin^{-1}(1)\right\}-\{0-0\}\right]$

$=\frac{4b}{a}\left[\frac{a^2}{2}.\frac{\pi}{2}\right]$

$=\pi ab\ sq\ units$

(ii) (a) The given equation of ellipse is $x^2+y^2=a^2$ ... ... (1)

This is a circle whose centre is (0,0) and radius ‘a’

Area of circle = 4(area of region 1st quadrant)

$=4\int_0^a y\,dx$

$= \int_0^a \sqrt{a^2-x^2}dx$

$\left[\because (1) \Rightarrow y=\pm\sqrt{a^2-x^2}\right]$

(But region OABO lies in 1st quadrant , y is positive)

$=4\int_0^a \sqrt{a^2-x^2}\,dx$

$=4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$=4\left[\left\{\frac{a}{2}(0)+\frac{a^2}{2}\sin^{-1}(1)\right\}-\{0-0\}\right]$

$=4\left[\frac{a^2}{2}.\frac{\pi}{2}\right]$

$=\pi a^2\ sq\ units$

(iii) (b) The given curve is $y=x^2$ .... (1)

And the given line is $y=4$ ... . (2)

$\therefore$ Required area = $2\int_0^4 x\ dy$

$=2\int_0^4 \sqrt{y}dy$

$=2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

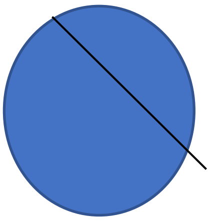
$=\frac{4}{3}\left[4^{\frac{3}{2}}-0\right]=\frac{4}{3}(8)=\frac{32}{3}\,sq\ units$

CHAPTER-8

APPLICATION OF INTEGRALS

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	A mirror in the shape of an ellipse $\frac{x2}{9}+\frac{y2}{4}$ =1 was hanging on the wall. Arun and his sister were playing with ball inside the house, even their mother refused to do so. All of a sudden, ball hit the mirror and got scratch in the shape of line represented by $\frac{x}{3}+\frac{y}{2}$= 1.  Based on the above information, answer the following question. a) Points of intersection of ellipse and the scratch are i) (0,2), (3,0) ii) (2,0), (0,3) iii) (2,3), (0,0) iv) (0,3), (3,0) b) The area of the smaller region bounded by the mirror and scratch is i) $3(\frac{\pi}{2}+1)$ sq. unit ii) $(\frac{\pi}{2}+1)$ sq. unit iii) $(\frac{\pi}{2}-1)$ sq. unit iv) $3(\frac{\pi}{2}-1)$ sq. unit c) The value of the integration $\int_{-1}^{0}(x+1)dx$ is i) $\frac{1}{2}$ ii) $\frac{2}{3}$ iii) $\frac{3}{4}$ iv) $\frac{1}{3}$ d) If the mirror is replaced by a circular mirror $x^2+y^2=1$ the new area of the mirror is i) 2π ii) π iii) π/4 iv) 1/π	4
2.	Pratik cut pizza with a knife .the shape of pizza is represented by the equation $x^2 + y^2 = 4$ and the sharpe edge of the knife represented by the straightline x=√3 y .	4

a) The point of intersection of the edge of knife (line) and the pizza shown in the figure are

i) $(1,\sqrt{3}), (-1,-\sqrt{3})$

ii) $(\sqrt{3},1), (-\sqrt{3},-1)$

iii) $(\sqrt{2},0),(0,\sqrt{3})$

iv) $(-\sqrt{3},1), (1,-\sqrt{3})$

b) Value of the area of the region bounded by circular pizza and edge of knife in 1st quadrant is

i) π/2 sq. unit

ii) π/3 sq. unit

iii) π/5 sq. unit

iv) πsq. Unit

c) Area of each slice of pizza when cut in to 4 pieces is

i) π. sq. Unit

ii) π/2 sq. unit

iii) 3π sq. unit

iv) 2π sq. unit

d) Area of whole pizza is

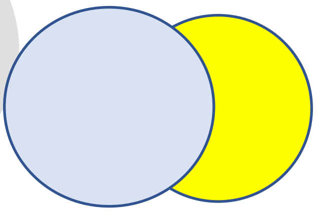
i) 3π sq. unit

ii) 2π sq. unit

iii) 5π sq. unit

iv) 4π sq. unit

3. In a partial solar eclipse when the moon and sun look overlapped as shown in the figure. The equation of the image of moon represented by the equation $(x-1)^2 + y^2 = 1$ and the image of sun is represented by the equation $x^2 + y^2 = 1$ — 4

a) The moon and sun meet each other at

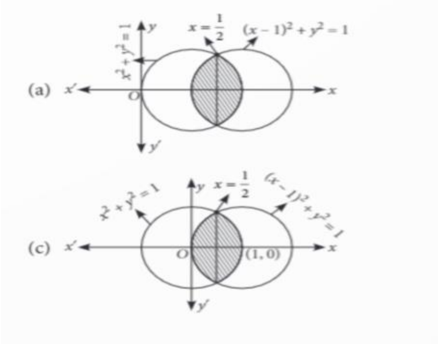
i) 1

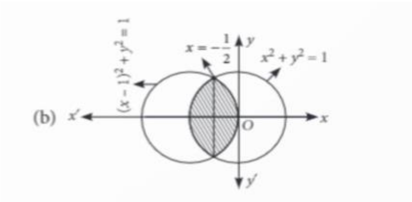
ii) ½

iii) 1/3

iv) ¼

b)





(d) None of these

c) Value of $\int_{1/2}^{1} \sqrt{(1-x^2)}\, dx$ is

i) $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$

ii) $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

iii) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

iv) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

d) Area of hidden portion of the lower circle is

i) $(\frac{2\pi}{3} + \frac{\sqrt{3}}{2})$ sq. unit

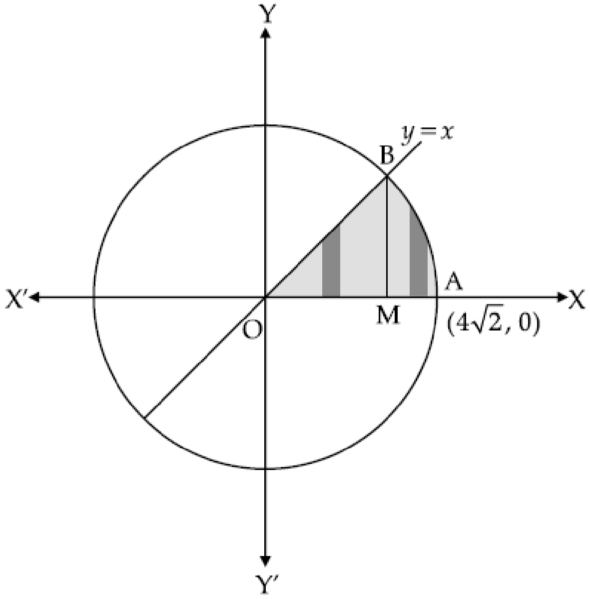
ii) $(\frac{\pi}{3} + \frac{\sqrt{3}}{8})$ sq. unit

iii) $(\frac{\pi}{3} - \frac{\sqrt{3}}{8})$ sq. unit

iv) $(\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$ sq. unit

4. Read the following text and answer the following questions on the basis of the same:

In the figure O (0, 0) is the centre of the circle. The line y = x meets the circle in the first quadrant at the point B. 4



(i) The equation of the circle is __________.

(a) $x^2 + y^2 = 4\sqrt{2}$ (b) $x^2 + y^2 = 16$

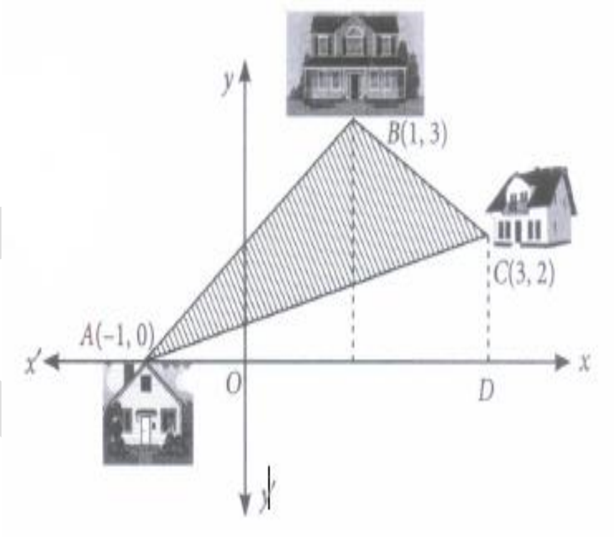
(c) $x^2 + y^2 = 32$ (d) None of these

(ii) The co-ordinates of B are __________.

(a) (1,1) (b) (2,2) (c) ($4\sqrt{2}, 4\sqrt{2}$) (d) (4,4)

(iii) Find the area of the shaded region.

5. Location of three houses of a society is represented by the points A(-1, 0), B(1, 3) and C(3, 2) as shown in figure. Based on the above information, answer the following question



(i) Equation of line AB is

(a) $y = \frac{3}{2}(x+1)$ (b) $y = \frac{3}{2}(x-1)$ (c) $y = \frac{1}{2}(x+1)$ (d) $y = \frac{1}{2}(x-1)$

(ii) Equation of line BC is

(a) $y = \frac{1}{2}x - \frac{7}{2}$ (b) $y = \frac{3}{2}x - \frac{7}{2}$

(c) $y = \frac{-1}{2}x + \frac{7}{2}$ (d) $y = \frac{3}{2}x + \frac{7}{2}$

(iii) Equation of line AC is

(a) $2y = x + 1$ (b) $2y = x - 1$

(c) $y = 2x + 1$ (d) $y = 2x - 1$

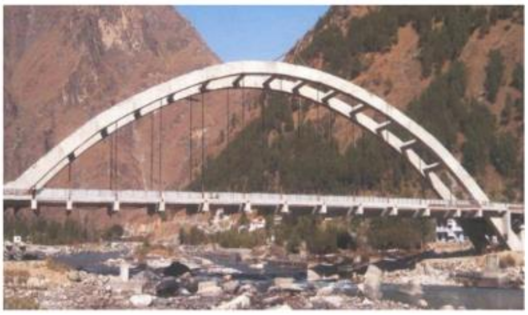
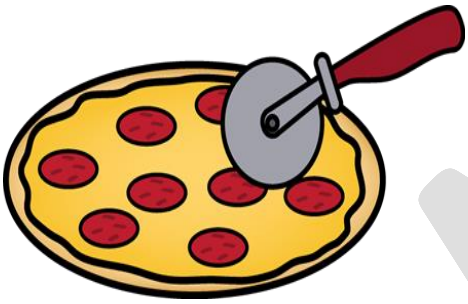
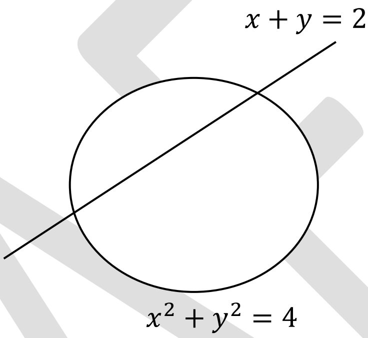
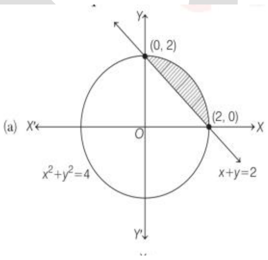
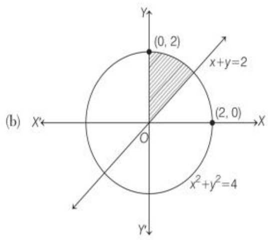
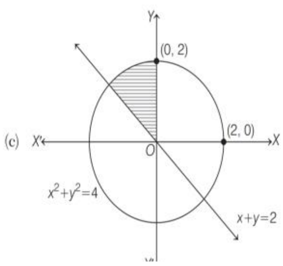
(iv) Find ar(ΔACD)

(a) 4sq. units (b) 8sq. units

(c)12sq. units (d) 2sq. units

4

6.	 Rajendra, a farmer had two sons and two daughters. He decided to divide his property among his sons and daughters .So he wrote a "WILL" about distribution of his property. According to his "WILL", he desired to give 3/5 th of the property to his sons in equal proportion, 1/3 rd to his daughters in equal proportion and rest to a charitable trust. After his death his "WILL" was opened and read out by the Advocate in the presence of all villagers. He stated in his WILL that my agriculture field is in the shape of triangle with vertices A(2,5), B(4,7) and C(6,2)and all will find the solution following questions based on the field. Those who will find the solution, will be given the stated share of my property (i) Find the equations of each side of triangular field. (ii) Find the area of field using integration.	4
7.	 Construction of airport is multi disciplinary project and in it involves the pooling of various engineering disciplines, agencies, experts, contractors, executives and the end users. Before entering into the real case studies of construction of runways and application of supply chain management technique it is essential to frame a construction plan and a map. The map shows parabolic entry curvatures in which distance between the legs of entry curvature is 60 feet and height of entry curvature is 15 feet. Based on the following information answer the following questions- (i) Find the equation of parabolic curvature. (ii) Find the area within the entry curvature	4
8.	The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.	4

	 Find the equation of the parabolic curve and area covered by the bridge and arch on the bridge.	
9.	Find the area of the region bounded by the curves $x^2 = y$, y=x+2 and x-axis, using integration.	4
10.	Rita was celebrating her birthday with her friends. She order a cake .She cut the cake with a knife. Cake was circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represented by a straight line given by $x + y = 2$   Based on the above information ,answer the following questions (i)The points of intersection of the edge of knife (line) and cake shown in the figure is (are) (a) $(0,2)$ *and* $(2,0)$ (b) $(0,1)$ *and* $(1,0)$ (c)$(1,2)$ *and* $(2,1)$ (d)$(3,1)$ *and* $(1,3)$ (ii) Which one of the following shaded portion represents the smaller area bounded by pizza and edge of knife in first quadrant (a)  (b)  (c)  (d) None of the above	4

(iii) Area of each piece of cake, when rita cut the cake into 4 equal pieces is

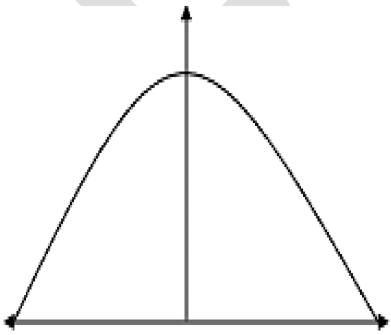
(a) $\frac{\pi}{2}$ $sq\ units$
(b) π $sq\ units$
(c) $\frac{\pi}{3}$ $sq\ units$
(a) 2π $sq\ units$

(iv) Area of whole cake is

(a) 4π $sq\ unit$
(b) 3π $sq\ unit$
(c) π $sq\ unit$
(a) $\frac{\pi}{2}$ $sq\ unit$

11. The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge, is 10 feet above the road at the middle of the bridge as seen in the figure. 4

Based on the information given above, answer the following questions:



(i) The equation of the parabola designed on the bridge is

(a) $x^2 = 250y$
(b) $x^2 = -250y$
(c) $y^2 = 250x$
(d) $y^2 = -250x$

(ii) The value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$

(a) $\frac{1000}{3}$
(b) $\frac{250}{3}$
(c) 1200
(d) 0

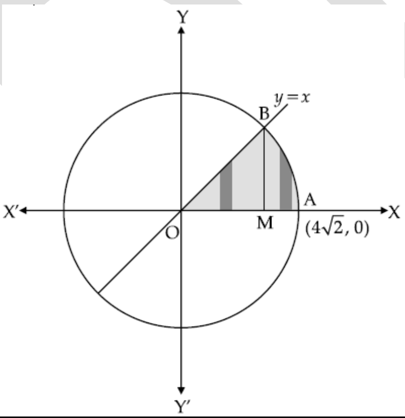
(iii) The integrand of the integral $\int_{-50}^{50} x^2 dx$ is __________ function.

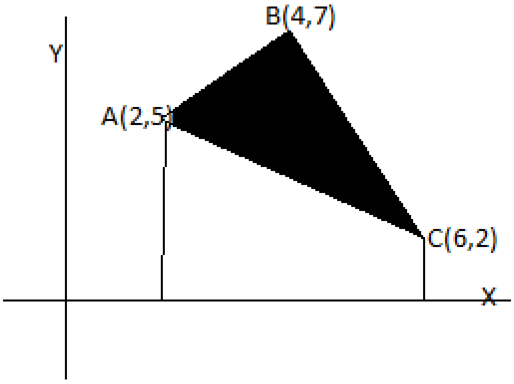
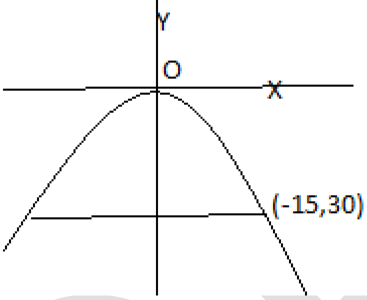
(a) Even
(b) Odd
(c) Neither odd nor even
(d) None

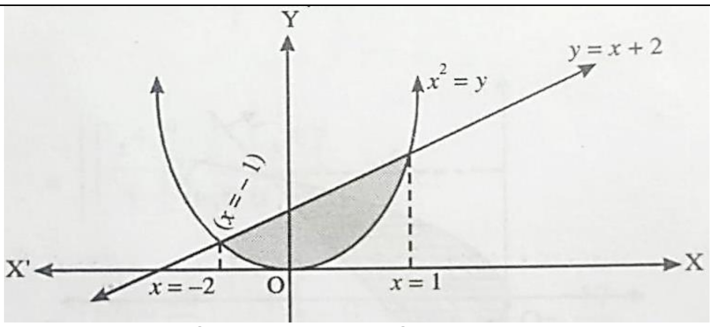
(iv) The area formed by the curve $x^2 = 250y$, x-axis,y=0 and y=10 is

	(a) $\frac{1000\sqrt{2}}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2\,000}{3}$ (d) 0	

ANSWERS:

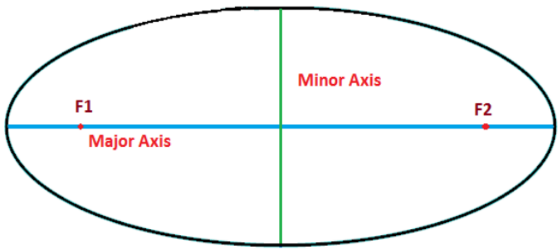
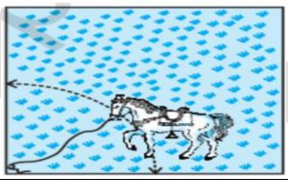
Q. NO	ANSWER	MARKS
1.	a) i b) iv c) i d) ii	4
2.	a) ii b) ii c) i d) iv	4
3.	a) ii b) iii c) iii d) iv	4
4.	(i) (c) (ii) (d) (iii) Given curve $y = x$-----(1) $x^2 + y^2 = 32$ --------(2)  Required area = Area of OABO = ar(ΔOBM)+ar(MABM) $= \frac{1}{2} \times OM \times BM + \int_4^{4\sqrt{2}} \sqrt{32 - x^2 dx}$ = 8+4π-8 = 4π	4
5.	(i) a (ii) c (iii) a (iv) a	4
6.	Figure of field is as follows	4

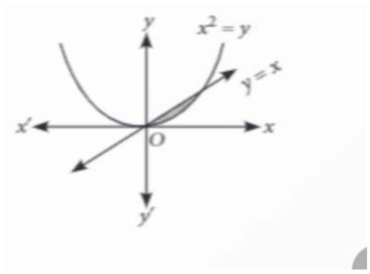
	 The equation of sides can be obtained using result $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$ Equation of AB: x-y+3=0 Equation of BC: 5x+2y=34 Equation of AC: 3x+4y-26=0 Area is given by $$A = \int_2^4 (x+3)dx + \int_4^6 \frac{34-5x}{2}dx - \int_2^6 \frac{26-3x}{4}dx$$ $$A = 7\ sq\ units$$	
7.	The figure is as follows  (i) Let equation of entry curvature $x^2 = -4ay$ Point (-15,30) satisfies the equation , so $(30)^2 = -(4a).(-15)$ gives a=15 So, the equation of entry curvature is $x^2 = -60y$ (ii) Area under entry curvature is given by $$A = 2\int_0^{30} \lvert y \rvert\, dx = 2\int_0^{30} \frac{x^2}{60}\, dx = 2/60[\frac{x^3}{3}]_0^{30} = 300\ sq\ units$$	4
8.	Equation of the curve is $X^2 = -250y$ Reqd. area = $\int_0^{10} x\, dy$ = $\int_0^{10} \sqrt{250\, y}\ dy$ = $\frac{1000}{3}$	4
9.	$x^2 = y$ ………(1) $y = x + 2$ …….(2) Solving eq. (1) and (2) $x^2 = x + 2$ x = -1, 2	4

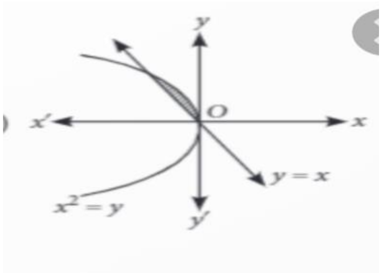
	 Reqd. area = $\int_{-1}^{2}(x+2)dx$ - $\int_{-1}^{2}x^2dx$ = 3 + $\frac{3}{2}$ = $\frac{9}{2}$ sq. units	
10.	(i) (a) $Given\ , x^2+y^2=4$ .....(1) $and\ x+y=2$ ......(2) Put the value of y from Eq. (2) in Eq. (1), we get $x^2+(2-x)^2=4$ $\Rightarrow x^2+4+x^2-4x=4$ $\Rightarrow 2x^2-4x=0$ $2x(x-2)=0$ $\Rightarrow x=0,2$ $when\ \ x=2\ \Rightarrow y=2$ $when\ \ x=2\Rightarrow y=0$ ∴Required points of intersection are (0,2) and (2,0) (ii) (a) (iii) (b) Given equation of circle is $x^2+y^2=4$ $(x-0)^2+(y-0)^2=(2)^2$ ∴ Radius of the circle is 2 units ∴Area of one forth cake =$\frac{1}{4}\pi(2)^2=\pi sq\ units$ (iv) (a) Area of whole cake = $\pi(2)^2=4\pi\ sq\ units$	4
11.	(i) –(b)$x^2=4ay$ $(x,y)=(50,-10)$ = $50^2=4a(-10)$ = $2500=-40a$ = $a=\frac{2500}{-40}$ = $a=-\frac{250}{4}$ ∴ $x^2=4\times\left(-\frac{250}{4}\right)y\Rightarrow x^2=-250y$ (ii) –(a) $\int_{-50}^{50}\frac{x^2}{250}dx$ ....$(even\ function)$ =2 $\int_{0}^{50}\frac{x^2}{250}dx$ = 2 × $\left[\frac{x^3}{250\times3}\right]_0^{50}$ = 2 × $\left(\frac{125000}{750}-0\right)$ =$\frac{1000}{3}$ (iii) –(a) even (iv) –(c) ∴Required area = 2 $\int_{0}^{10}xdx$	4

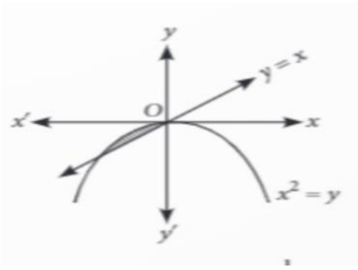
	$= 2\int_0^{10} \sqrt{250y}dy$ $=10\sqrt{10}\left(\frac{2}{3}y^{\frac{3}{2}}\right)_0^{10}$ $=\frac{20\sqrt{10}}{3} \times 10\sqrt{10}$ $= \frac{2000}{3}$	

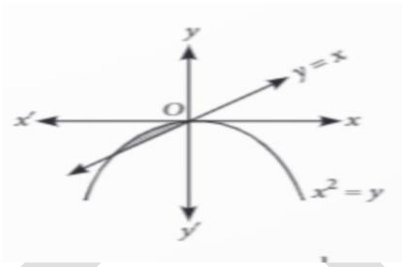
CHAPTER-8
APPLICATION OF INTEGRALS
05 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Find the area of the circle $x^2+y^2=16$ exterior to the parabola.	5
2.	Find the area of the region lying above X-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.	5
3.	Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.	5
4.	Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$	5
5.	Using integration, find the area of region bounded by the triangle whose vertices are (-1, 0),(1, 3) and (3, 2).	5
6.	Show that the area cut off by a parabola in first quadrant and ordinate is one third of the corresponding rectangle formed by that ordinate and its distance from the vertex using integration.	5
7.	Find the area of the region bounded by the curve y = tan x, tangent to the curve at point at x = π/4 and the x-axis using integration.	5
8.	A particle is moving as a elliptical curve, whose horizontally maximum distance is 8 km and vertically maximum distance is 6 km.  Then find the area covered by the particle.	5
9.	**A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see Fig.).** **Find the area of that part of the field in which the horse can graze by using integration.** 	5
10.	Consider the following equation of curves $x^2 = y \, and \, y = x$ On the basis of above information, answer the following questions (i) The point(s) of intersection of both the curves is (are) (a) $(0,0), (2,2)$ (b) $(0,0), (1,1)$ (c) $(0,0), (-2,-2)$ (d) $(0,0), (-1,-1)$ (ii) Area bounded by the curves is represented by which of the following graphs?	5

(a)


(b)


(c)


(d)


(iii) The value of the integral $\int_0^1 x\,dx$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(iv) The value of the integral $\int_0^1 x^2\,dx$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(v) The value of area bounded by the curves $x^2 = y \; and \; y = x$ is

(a) $\frac{1}{6} sq.unit$ (b) $\frac{1}{3} sq.unit$

(c) $\frac{1}{2} sq.unit$ (d) $1 sq.unit$

11. Location of three branches of a bank is representedby the three points $A(-2,0), B(1,4) and\ c(2,3)$ asshown in figure. **5**

(i) Equation of line AB is

(a) $y = \frac{4}{3}(x+2)$

(b) $y = \frac{4}{3}(x+1)$

(c) $y = \frac{4}{5}(x+2)$

(d) $y = \frac{4}{5}(x+1)$

(ii) Equation of line BC is

(a) $y = x + 5$

(b) $y = -x + 5$

(c) $y = x + 4$

(d) $y = -x + 4$

(iii)Area of region ABCD is4 $sq\ units$

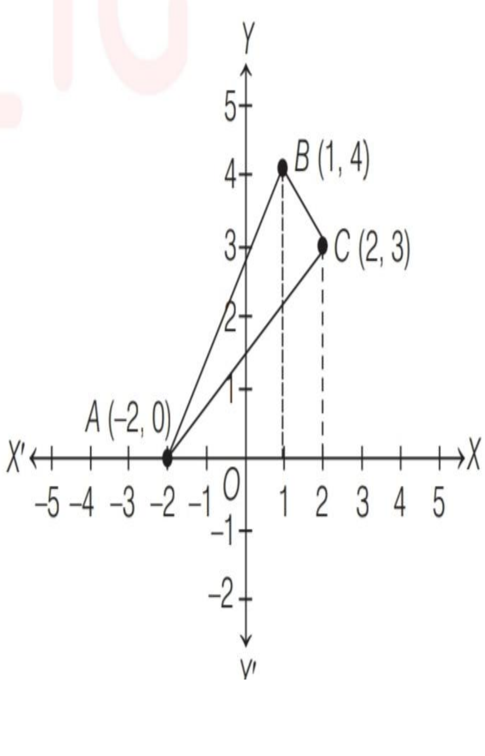
(a) $19\ sq\ units$

(b) $\frac{19}{2}\ sq\ units$

(c) $17\ sq\ units$

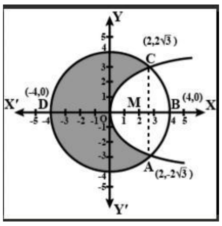
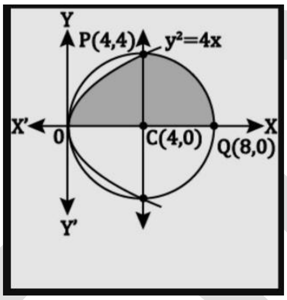
(d) $6\ sq\ units$

(iv)Area of $\Delta ADC$is



	(a) $3\ sq\ units$ (b) $4\ sq\ units$ (c) $6\ sq\ units$ (d) $5\ sq\ units$ (v) Area of ΔABC is (a) $7\ sq\ units$ (b) $\frac{3}{2}\ sq\ units$ (c) $5\ sq\ units$ (d) $\frac{7}{2}\ sq\ units$	

ANSWERS:

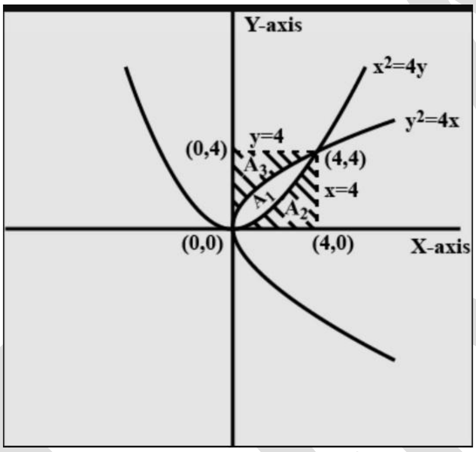
Q. NO	ANSWER	MARKS
1.	 The given equation are $x^2 + y^2 = 16$ and $y^2 = 6x$ Area bounded by the circle and the parabola = = 2[area (OADO) + area(ADBA)] $=2 [\int_0^2 \sqrt{6x}\, dx + \int_2^4 \sqrt{16-x^2}\, dx]$ by finding the integration we will get $\frac{4}{3}[4\pi + \sqrt{3}]$ sq.unit. area of the circle $\pi r^2 = \pi 4^2 = 16\pi$ required Area = $16\pi - \frac{4}{3}[4\pi + \sqrt{3}]$ $= \frac{4}{3}[8\pi - \sqrt{3}]$ Ans.	5
2.	 Given: $Y^2=4x$ x2+y2=8x $\Rightarrow x2-8x+y2=0$ $\Rightarrow x2-2\times4\times x+y2=0$ $\Rightarrow x2-2\times4\times x+42-42+y2=0$ $\Rightarrow (x-4)2+y2=42$ So, circle has centre 4,0 & radius=4 Equation of circle is x2+y2=8x Putting y2=4x $\Rightarrow$x2+4x=8x $\Rightarrow x2-4x=0$ $\Rightarrow x(x-4)=0$ $\Rightarrow$x=0 & x=4 For x=0	5

<table>
<tr><td></td><td>Y^2 =4x
=y =0
Point is (0,0)

For x=4
Y^2 =4x =16
Y =±4
So, point is (4,4)
And (4,−4)

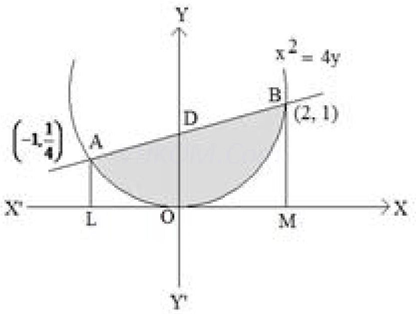
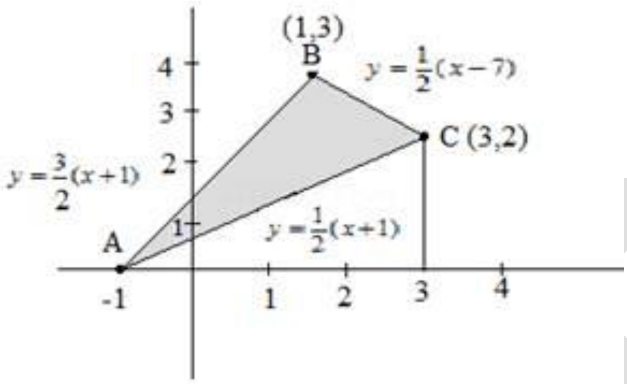
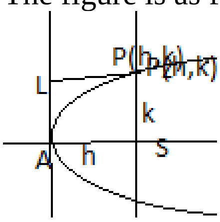
Since, point P is in 1st quadrant, so, the coordinates of P = (4,4)

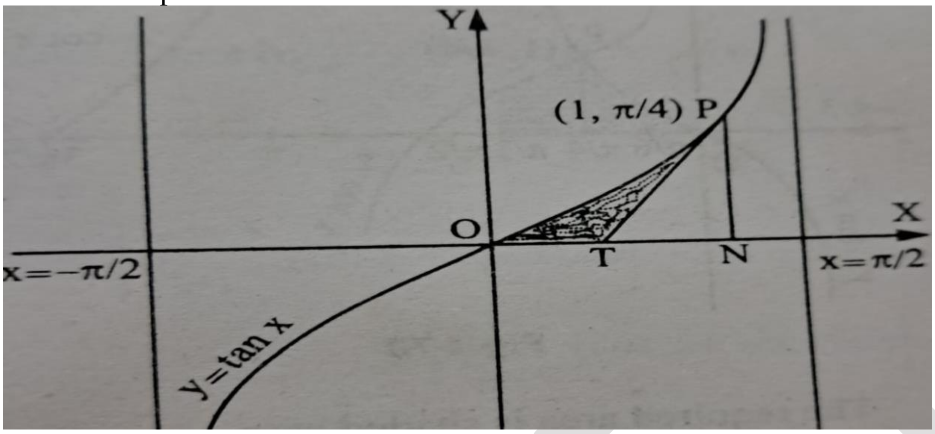
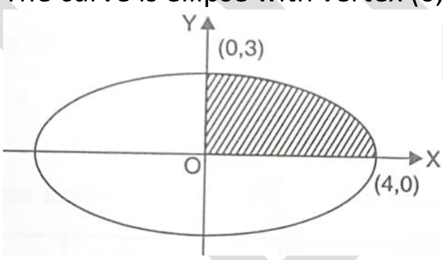
Equation of the curves in first quadrant is
Parabola: y=2√x
Circle: (x−4)2+y2=42
⇒y=√42−(x−4)2
Area Required=AreaOPCO+AreaPCQP
=∫40y1 dx+∫84y2dx

where y1=2√x and y2=√42−(x−4)2
so by solving we gwt the required area $\frac{4}{3}(8+3\pi)$. ANS.</td><td></td></tr>
<tr><td>3.</td><td>

To prove A1= A2= A3 = $\frac{A}{3}$ whetre A is the area of the square.
A=4*4= 16 sq. unit
A1 = $\int_0^4 \frac{x^2}{4}dx = \frac{16}{3}$ sq. unit
A2 = $\int_0^4 \frac{y^2}{4}dy = \frac{16}{3}$ sq. unit

A3= A - (A1+A2)
=16- 2*$\frac{16}{3}$
=$\frac{16}{3}$ sq. unit
So A1=A2=A3=$\frac{16}{3}$ sq. unit.</td><td>5</td></tr>
<tr><td>4.</td><td>$x^2 = 4y$ -------(1)
$x = 4y - 2$ -----(2)</td><td>5</td></tr>
</table>

	 Req. area $= \int_{-1}^{2} \frac{1}{4}(x+2)\,dx - \frac{1}{4}\int_{-1}^{2} x^2 dx$ $= \frac{9}{8}\, sq\ unit$	
5.	 A (-1, 0) B (1, 3) C (3, 2) Equation of AB $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ $y - 0 = \frac{3-0}{1+1}(x+1)$ $y = \frac{3}{2}(x+1)$ Similarly Equation of BC $y = \frac{-1}{2}(x-7)$ Equation of AC $= \frac{1}{2}(x+1)$ $Area\ \Delta ABC = \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3} \frac{1}{2}(x-7)\,dx$ $-\int_{-1}^{3} \frac{1}{2}(x+1)\,dx$ $= 4 sq.\ unit$	5
6.	The figure is as follows  The equation of parabola is $y^2 = 4ax$ Let ordinate be drawn through P(h,k)	5

	Then $k^2 = 4ah$ or $k=2\sqrt{(ah)}$ So area of triangle = k.h = 2h $\sqrt{(ah)}$ Now area of triangle = $\int_0^h y\,dx = \int_0^h 2\sqrt{a\sqrt{x}dx} = \frac{1}{3}\left(2h\sqrt{ah}\right)$ = one third of area of triangle	
7.	The required area is as shown in shaded portion bounded by curve y = tan x , tangent PT at P and part OT of x-axis  Now $\frac{dy}{dx} = sec^2 x = 2$ at x = π/4 Also y = tan π/4 = 1 at point P(π/4 , 1) Equation of tangent is y – 1 = 2(x – π/4) y = 2x + 1 -π/2 When y = 0 then x = π/2 – ½ = OT So, TN = ON – OT = π/4 - π/4 +1/2 = ½ Required area is given by Area OPNO - Area of ΔPTN $= \int_o^{\pi/4} \tan x\,dx - \frac{1}{2}\,TN.PN$ = log√2 – ¼ = 1/2 (log2 – ½)	5
8.	Equation of the curve is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ The curve is ellipse with vertex (0, 0)  The area of the region bounded by the given ellipse = 4 × Area of the ellipse in the first quadrant Reqd. area = 4 $\int_0^4 y\,dx$ = 4 $\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx$ = 12 π sq. units	5
9.	Equation of the curve is $x^2 + y^2 = 25$ Reqd. area = area of the circle in the first quadrant. = $\int_0^5 y\,dx$ = $\int_0^5 \sqrt{25 - x^2}\,dx$	5

		$=\frac{25\pi}{4}$ sq. units	
10.	(i)	(b)We have $x^2 = y \ldots\ldots (1) and \quad x = y \ldots\ldots..(2)$ From eq (1) and (2), $x^2 = x \Rightarrow x^2 - x = 0$ $\Rightarrow x(x-1) = 0 \Rightarrow x = 0,1$ $from\ Eq.(2) y = 0,1$ ∴Required points of intersection are (0,0) ,(1,1).	5
	(ii)	(a)	
	(iii)	(c) $\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$	
	(iv)	(b) $\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$	
	(v)	(a) Required area $=\int_0^1 x dx - \int_0^1 x^2 dx$ $=\frac{1}{2} - \frac{1}{3} = \frac{1}{6} sq\ units$	
11.	(i)	(a) Equation of line AB is given by $(y-0) = \frac{4-0}{1+2}(x+2) \Rightarrow y = \frac{4}{3}(x+2)$	5
	(ii)	(b) Equation of line BC is given by $(y-4) = \frac{3-4}{2-1}(x-1) \Rightarrow y = -x+5$	
	(iii)	(b) Area of the region ABCD = Area of ΔABE + Area of region BCDE $=\int_{-2}^{1} \frac{4}{3}(x+2) dx \int_1^2 (-x+5) dx$ $=\frac{4}{3}\left[\frac{x^2}{2} + 2x\right]_{-2}^{1} + \left[-\frac{x^2}{2} + 5x\right]_1^2$ $=\frac{4}{3}\left[\frac{1}{2} + 2 - 2 - 2 + 4\right] + \left[-2 + 10 + \frac{1}{2} - 5\right]$ $=\frac{4}{3}.\frac{9}{2} + \left(\frac{1}{2} + 3\right)$ $=6 + \frac{7}{2} = \frac{19}{2}\ sq\ units$	
	(iv)	(c) Equation of line AC is given by $(y-0) = \frac{3-0}{2+2}(x+2) \Rightarrow y = \frac{3}{4}(x+2)$ Area of $\Delta ADC = \int_{-2}^{2} \left(\frac{3}{4}(x+2)\right) dx$ $=\frac{3}{4}\left[\frac{x^2}{2} + 2x\right]_{-2}^{2} \Rightarrow \frac{3}{4}(2+4-2+4) \Rightarrow \frac{3}{4}.8 = 6\ sq\ units$	
	(v)	(d) Area of $\Delta ABC = (iii) - (iv)$ $= \frac{19}{2} - 6 = \frac{7}{2} sq\ units$	

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